**Supplemental appendix S1: Derivation of standardized densities, *vjx***

The second-order approximation of the low-density metacommunity-scale growth rate of an invader requires first-order approximations of the relative densities of the resident and invader, *vjx*. For ease of calculation, I standardize the relative densities to have a mean of zero, e.g. . Those expressions are derived here.

*Derivation of urx*

The standardized density of the resident is defined as . By definition, the resident density is at equilibrium, . Thus,

 .

 Eqn. A.1 states that the standardized density of the resident in patch *x* is equal to the standardized supply rate in that patch.

*Derivation of uix*

The standardized density of the invader is . Unfortunately, the general expression for is unknown. My strategy is to estimate *uix(t)* by finding an expression for, then solve this expression for the equilibrium values,**. This method of finding *uix(t)* assumes that the standardized relative density of the invader reaches its equilibrium much faster than invader’s density.

The rate of change of the invader is in patch *x* is

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This can be written in terms of standardized densities and supply rates, , to give

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A first-order Taylor expansion around , after combining terms, results in

 .

Taking a spatial mean of both sides gives

 .

By definition, . Taking the time derivative of each side, applying the quotient rule to the right hand side and simplifying gives

 ,

which yields the equilibrium solution

 .