

1 **Supporting Information**

2 **Estimating and comparing microbial diversity in the presence of sequencing errors**

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6 **Supplemental Text S2. Simulation results based on six species abundance models**

7 To investigate the performance of the proposed singleton count derived in Equation (5) and the
8 diversity estimator in Equation (7) of the main text, we carried out simulations by generating data
9 sets from various species abundance models. Here we report the results from six representative
10 models. In each model, we fixed the number of species at $S = 2000$ to mimic the taxa richness of
11 microbial communities.

12 The functional forms or distributions for species' relative abundances (p_1, p_2, \dots, p_S) are
13 given below, whereby c is a normalizing constant such that $\sum_{i=1}^S p_i = 1$. When species abundances
14 were simulated from a distribution (Models 2, 3 and 4), we first generated a set of 2000 random
15 variables, which we regarded as fixed parameters in the simulation. In each model, we also give
16 the CV (which is the ratio of the standard deviation over the mean) of (p_1, p_2, \dots, p_S) . The CV
17 value quantifies the degree of heterogeneity among the species' relative abundances
18 (p_1, p_2, \dots, p_S) . When all abundances are equal, $CV = 0$. A larger value of CV indicates a higher
19 degree of heterogeneity among abundances. In the following description, $S = 2000$ for all models.

20
21 **Model 1.** A homogeneous model with $p_i = 1/S$ and $S = 2000$. This is the model with no
22 heterogeneity among species relative abundances ($CV = 0$).

23 Model 2. A random uniform model with $p_i = ca_i$, where (p_1, p_2, \dots, p_S) is a random sample from a
24 uniform (0, 1) distribution. (CV = 0.57).

25 Model 3. A broken-stick model with $p_i = ca_i$, where (a_1, a_2, \dots, a_S) is a random sample from an
26 exponential distribution. Equivalently, (p_1, p_2, \dots, p_S) follows a Dirichlet distribution with
27 parameter 1 (CV = 0.99).

28 Model 4. A log-normal model with $p_i = ca_i$, where (a_1, a_2, \dots, a_S) is a random sample from a
29 log-normal distribution with mean $\mu = 0$, and variance $\sigma^2 = 1$ (CV = 1.96).

30 Model 5. A Zipf-Mandelbrot model with $p_i = c/(i+5)$, $i = 1, 2, \dots, S$ (CV = 3.07).

31 Model 6. A power-decay model with $p_i = c/i^{0.9}$, $i = 1, 2, \dots, S$ (CV = 5.03).

32
33 For each given model, we considered a range of sample sizes ($n = 2000$ to 10000 in
34 increments of 1000). Then for each combination of abundance model and sample size, 1000
35 simulated data sets were generated from the abundance model. Two types of data were generated:
36 (i) Data without sequencing error (i.e., data with the true number of singletons): individuals were
37 randomly selected from a given model and their species identities were correctly recorded.
38 (ii) Spurious data with a sequencing error rate of 10% (data with spurious singletons): individuals
39 were randomly selected from a given model, but there was a probability of 10% that each sampled
40 individual was misclassified as a new species and thus became a spurious singleton. This was used
41 to mimic the sequencing error with an error rate of 10% for each detected individual to be
42 misclassified as a spurious singleton.

43 For each model, we display four sub-plots in Supplementary Fig. S1: In Panel (a), we show
44 the plots of the average values of four singleton counts as a function of the sample size that was
45 used in data generation. The four singleton counts include the true singleton count generated from

46 the data without sequencing error, the spurious singleton count generated from the data with
47 sequencing error, the adjusted singleton count based on Equation (5), and the count obtained from
48 the ratio-based method of Bunge et al. (2014) and Willis & Bunge (2015) through the R package
49 “breakaway”, available from CRAN (Comprehensive R Archive Network). All values were
50 averaged over 1000 simulation trials under the six species abundance models. All plots in Panels
51 (a) were also shown in Fig. 1 of the main text; see the main text for the comparisons of the
52 performances of the four singleton counts.

53 Under each model, Panels (b), (c) and (d) compare the true diversity (Equation 1 in the main
54 text) and the estimated asymptote of diversity (Equation 7 in the main text). There are two
55 estimated diversities, respectively calculated from the spurious data and from the adjusted data. As
56 described in the main text, the “adjusted data” refer to those with the observed singleton count
57 being replaced by the estimated count computed from Equation (5) of the main text.

58 Panel (b) for each model shows the plots of the true species richness and the average values
59 (over 1000 simulation trails) of the Chao1 estimator for the spurious data, the Chao1 estimator for
60 the adjusted data, as well as the species richness estimator via the ratio-based method described
61 above. It is clear that the Chao1 estimator for the spurious data severely overestimates the true
62 species richness. By contrast, the Chao1 estimator for the adjusted data reduces most of the
63 positive bias and works well for all models, although negative bias exists with the magnitude of
64 the bias increasing with CV value. While the ratio-based method also works when CV value is
65 relatively low (Model 1 to Model 4), the ratio-based species richness estimates exhibit large
66 positive bias when the CV value is relatively high (Model 5 and Model 6).

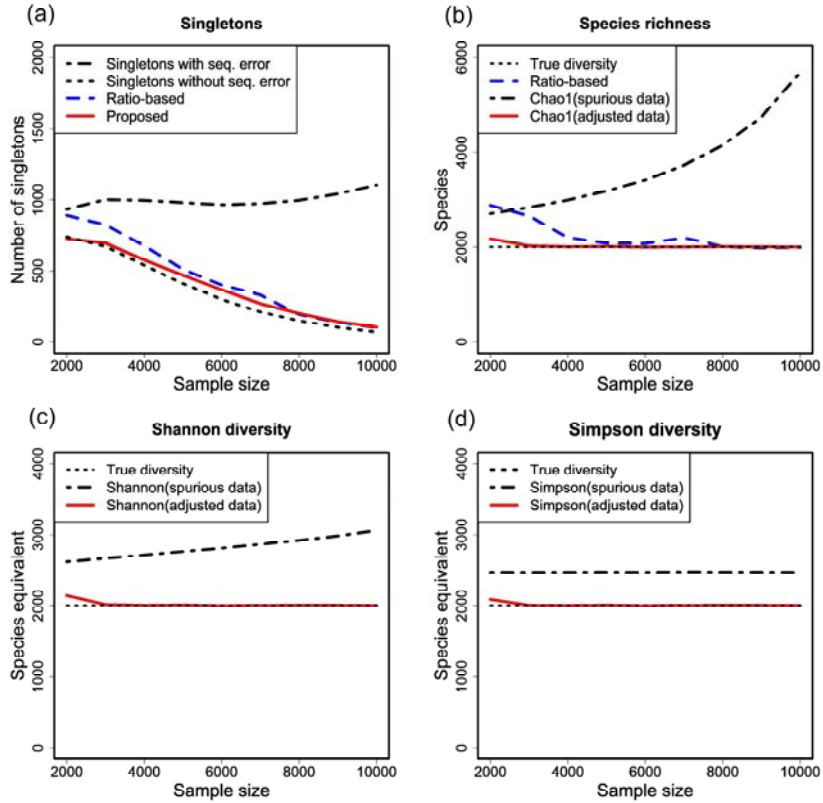
67 In Panel (c), we show the plots of the true Shannon diversity and the average values (over
68 1000 simulation trails) of the estimated Shannon diversity for the spurious data and for the
69 adjusted data. The corresponding plots for Simpson diversity are displayed in Panel (d). Although

70 the simulation results in Panel (b) of each model demonstrate that the species richness estimation
71 is seriously inflated or affected by spurious singleton counts, the effect on Shannon diversity is
72 moderate and the effect on Simpson diversity is weak, as shown in Panel (c) and Panel (d) in each
73 model). Under each model, both the estimated Shannon and Simpson diversities computed from
74 spurious data overestimate the true diversities, although the bias is not as severe as it is for species
75 richness. Our estimated Shannon and Simpson diversities for the adjusted data exhibit very low
76 bias (when sample size is small) or are nearly unbiased (when sample size is sufficiently large) for
77 all models.

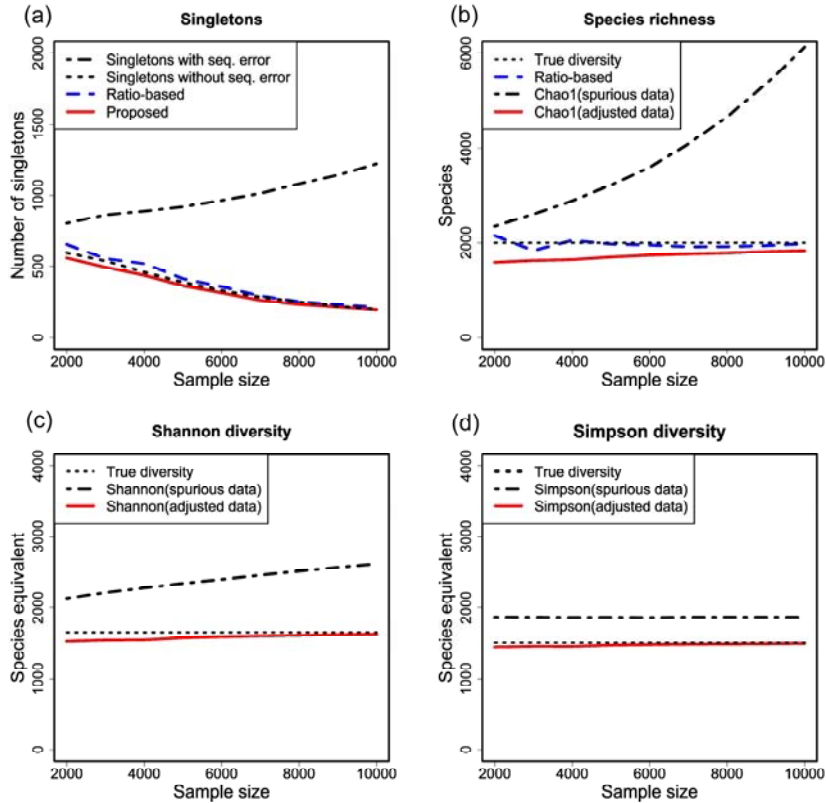
78 In summary, our estimated asymptotes of diversities presented in Equation (7) of the main
79 text based on the adjusted data greatly remove the positive biases due to spurious singletons. When
80 there are sequencing errors, our procedure always leads to better results; when there are no
81 sequencing errors, our results differ from those based on the true data only to a limited extent.
82 Therefore, our proposed estimator of singleton count can be used to detect the quality of the
83 observed singleton count. This also reveals that whenever singletons are uncertain or in doubt, it is
84 worth applying our estimator of singleton count in diversity analysis and statistical inferences.

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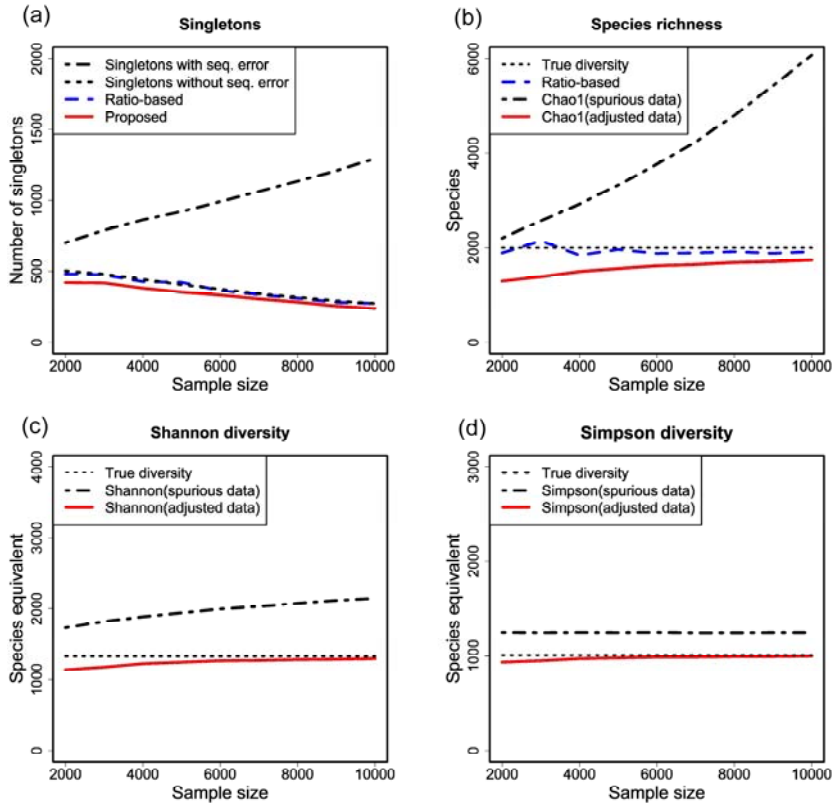
Model 1: Homogeneous model (CV=0)



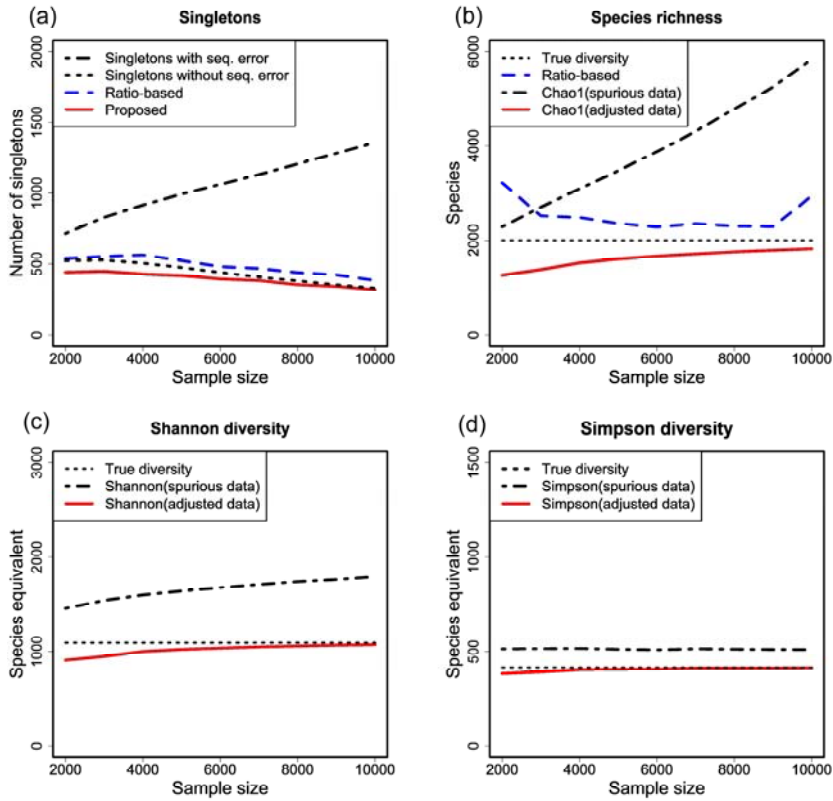
Model 2: random uniform model (CV=0.57)



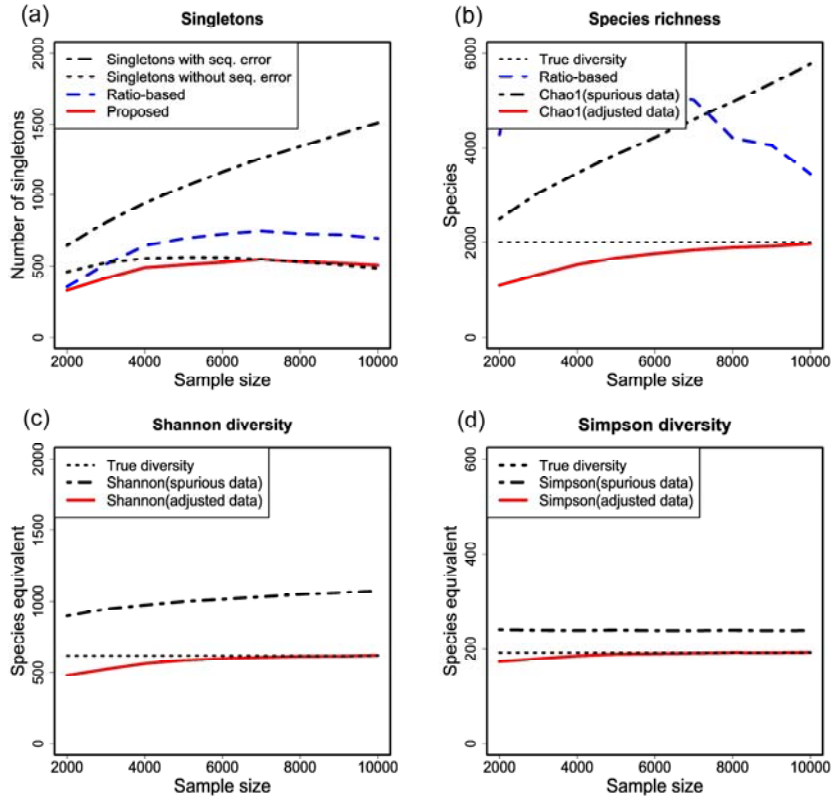
Model 3: broken-stick model (CV=0.99)



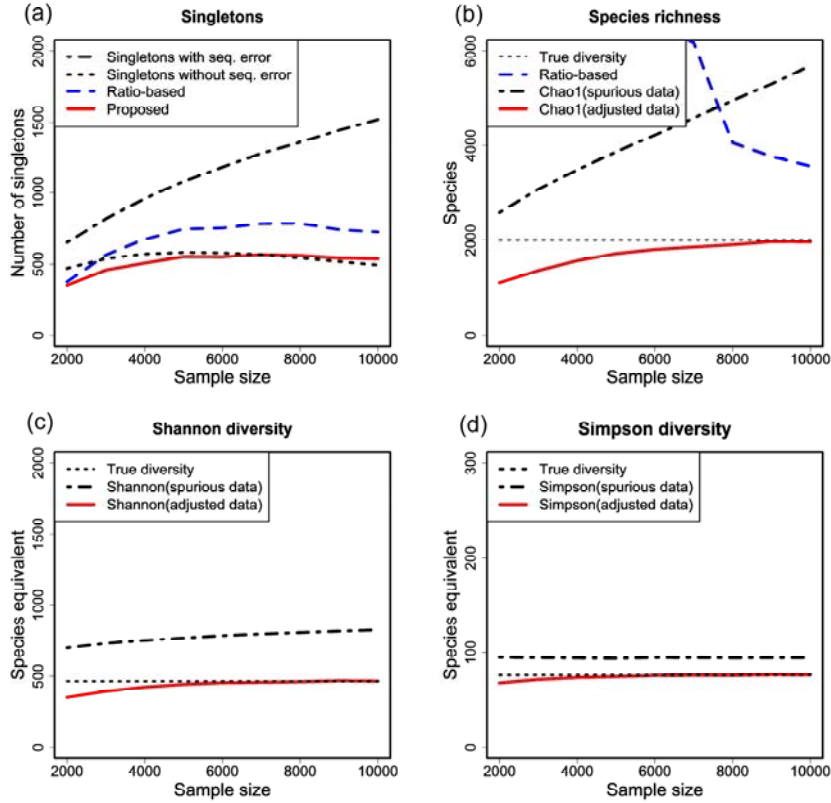
Model 4: log-normal model (CV=1.96)



Model 5: Zipf-Mandelbrot model (CV=3.07)



Model 6: power decay model (CV=5.03)



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90 **Fig S1. Plots of simulation results.** Under each model, there are four panels.

91 Panel (a) compares the average values of four singleton counts: the true singleton count generated
92 from the data without sequencing error, the spurious singleton count generated from the data
93 with sequencing error, the adjusted singleton count based on Equation (5), and the count
94 obtained from the ratio-based method of Bunge et al. (2014) and Willis & Bunge (2015) through
95 the R package “breakaway”, available from CRAN (Comprehensive R Archive Network). All
96 values represent the average values over 1000 simulation trials under six species abundance
97 models.

98 Panel (b) compares the true species richness, and the average values (over 1000 simulation trials)
99 of the Chao1 estimator for the spurious data, the Chao1 estimator for the adjusted data, and the
100 species richness estimator obtained from the ratio-based approach.

101 Panel (c) compares the true Shannon diversity and the average values (over 1000 simulation trials)
102 of the estimated Shannon diversity for the spurious data and for the adjusted data.

103 Panel (d) compares the true Simpson diversity and the average values (over 1000 simulation trials)
104 of the estimated Simpson diversity for the spurious data and for the adjusted data.

105 Note the scale of the Y-axis in each model may be different in the four panels due to different
106 ranges of diversity.

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108 **References**

- 109 Bunge J, Willis A, Walsh F. 2014. Estimating the number of species in microbial diversity studies.
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111 10.1146/annurev-statistics-022513-115654.
- 112 Willis A, Bunge J. 2015. Estimating diversity via frequency ratios. *Biometrics*, early online version.
113 DOI: 10.1111/biom.12332.