# Appendix A: Proof of Theorem

*Proof.* First, we show that Equation (9) and (8) are valid for any internal node $x_i$ when calculated by post-order traversal. When we reach $x_i$ it can be an internal node or the root node. If $x_i$ is an internal node, we distinguish between the children being leaf nodes or internal nodes. Images for Figure 1 were taken from Wikipedia.

1) *$x_{L(i)}$ and $x_{R(i)}$ are leaf nodes.* Then Equation (4) gives, (using $\phi_{L(i)} = m_{L(i)}$, $\phi_{R(i)} = m_{R(i)}$ for child nodes $x_L(i)$ and $x_R(i)$, and the definition of $m_i$ and $\rho_i$)

$$\begin{aligned}
\phi_i &= l_i\phi_{L(i)} + r_i\phi_{R(i)} + p_i\phi_{P(i)} \\
&= l_i m_{L(i)} + r_i m_{R(i)} + p_i\phi_{P(i)} \\
&= \frac{l_{L(i)}m_{L(i)} + r_{R(i)}m_{R(i)}}{1 - l_{L(i)}0 - r_{R(i)}0} + \frac{p_i}{1 - l_{L(i)}0 - r_{R(i)}0}\phi_{P(i)} \\
&= m_i + \rho_i\phi_{P(i)}.
\end{aligned}$$

2) *$x_{L(i)}$ is a leaf and $x_{R(i)}$ an internal node.* Then, $\phi_{R(i)} = m_{R(i)} + \rho_{R(i)}\phi_i$ (by post-order traversal) and $\phi_{L(i)} = m_{L(i)}$. Then Equation (4) gives,

$$\begin{aligned}
\phi_i &= l_i\phi_{L(i)} + r_i\phi_{R(i)} + p_i\phi_{P(i)} \\
&= l_i m_{L(i)} + r_i(m_{R(i)} + \rho_{R(i)}\phi_i) + p_i\phi_{P(i)}
\end{aligned}$$

grouping $\phi_i$ and divide by $1 - 0 - r_i\rho_{R(i)}$ gives

$$\begin{aligned}
lat_i &= \frac{l_{L(i)}m_{L(i)} + r_{R(i)}m_{R(i)}}{1 - l_{L(i)}0 - r_{R(i)}\rho_{R(i)}} + \frac{p_i}{1 - l_{L(i)}0 - r_{R(i)}\rho_{R(i)}}\phi_{P(i)} \\
&= \frac{l_{L(i)}m_{L(i)} + r_{R(i)}m_{R(i)}}{1 - l_{L(i)}\rho_{L(i)} - r_{R(i)}\rho_{R(i)}} + \frac{p_i}{1 - l_{L(i)}\rho_{L(i)} - r_{R(i)}\rho_{R(i)}}\phi_{P(i)} \\
&= m_i + \rho_i\phi_{P(i)}.
\end{aligned}$$

3) *$x_{L(i)}$ is an internal node and $x_{R(i)}$ a leaf node.* By symmetry, case 2 holds.

4) *$x_{L(i)}$ and $x_{R(i)}$ are both internal nodes.* Then, $\phi_{L(i)} = m_{L(i)} + \rho_{L(i)}\phi_i$ and $\phi_{R(i)} = m_{R(i)} + \rho_{R(i)}\phi_i$(by post-order traversal). Now Equation (4) gives,

$$\begin{aligned}
\phi_i &= l_i\phi_{L(i)} + r_i\phi_{R(i)} + p_i\phi_{P(i)} \\
&= l_i(m_{L(i)} + \rho_{L(i)}\phi_i) + r_i(m_{R(i)} + \rho_{R(i)}\phi_i) + p_i\phi_{P(i)}
\end{aligned}$$

grouping $\phi_i$ and divide by $1 - l_i\rho_{L(i)} - r_i\rho_{R(i)}$ gives

$$\begin{aligned}
lat_i &= \frac{l_{L(i)}m_{L(i)} + r_{R(i)}m_{R(i)}}{1 - l_{L(i)}\rho_{L(i)} - r_{R(i)}\rho_{R(i)}} + \frac{p_i}{1 - l_{L(i)}\rho_{L(i)} - r_{R(i)}\rho_{R(i)}}\phi_{P(i)} \\
&= m_i + \rho_i\phi_{P(i)}.
\end{aligned}$$

If $x_i$ is the root Equation (8) follows from a similar argument, but with $\rho_i = 0$.

From equations Equations (9) and (8) together it follows that the mean can be calculated by post-order traversal to provide information for the root location (8) to be calculated, followed by a pre-order traversal to calculate locations of internal nodes using (9).

Complexity: it is easy to see the algorithm is $O(n)$ since it takes one post-order traversal sending $2n - 2$ messages up and one pre-order traversal calculating locations for $n - 1$ internal nodes. Each message and latitude calculation is $O(1)$. $\square$

## Appendix B: HBV data

| Genbank | year | location | Genbank | year | location | Genbank | year | location |
|---|---|---|---|---|---|---|---|---|
| AB026814 | 1998 | Japan | AB198076 | 2001 | China | AJ748098 | 2002 | Vietnam |
| AB033550 | 1988 | Japan | AB198077 | 2001 | China | AY341335 | 2003 | Greece |
| AB033554 | 1985 | Indonesia | AB198078 | 2001 | China | AY641558 | 2003 | Korea |
| AB033555 | 1984 | Indonesia | AB198079 | 2001 | China | AY641559 | 2003 | Korea |
| AB033556 | 1985 | Japan | AB198080 | 2001 | China | AY641560 | 2003 | Korea |
| AB049609 | 1996 | Japan | AB198081 | 2001 | China | AY641561 | 2003 | Korea |
| AB049610 | 1996 | Japan | AB198082 | 2001 | China | AY641562 | 2003 | Korea |
| AB106564 | 1999 | Ghana | AB198083 | 2001 | China | AY641563 | 2003 | Korea |
| AB109475 | 2001 | Japan | AB198084 | 2001 | China | AY721605 | 2004 | Turkey |
| AB109476 | 1997 | Japan | AB205010 | 1994 | Japan | AY721606 | 2004 | Turkey |
| AB109477 | 1997 | Japan | AB205118 | 2001 | Japan | AY721607 | 2004 | Turkey |
| AB109478 | 1998 | Japan | AB205119 | 2000 | Japan | AY721608 | 2004 | Turkey |
| AB109479 | 1997 | Japan | AB205120 | 2000 | Japan | AY721609 | 2004 | Turkey |
| AB110075 | 2001 | Japan | AB205122 | 2001 | Vietnam | AY721611 | 2004 | Turkey |
| AB111121 | 1998 | Japan | AB205123 | 2002 | China | AY721612 | 2004 | Turkey |
| AB111125 | 2001 | Japan | AB205124 | 2003 | Japan | AY738143 | 1997 | Germany |
| AB111946 | 1998 | Vietnam | AB205125 | 2001 | Vietnam | AY738147 | 1998 | DR Congo |
| AB112063 | 1998 | Vietnam | AB205126 | 2001 | Japan | AY796030 | 2004 | Turkey |
| AB112065 | 1998 | Vietnam | AB205127 | 2000 | Russia | AY796031 | 2004 | Turkey |
| AB112066 | 1999 | Myanmar | AB205128 | 2000 | Russia | D23680 | 1991 | Japan |
| AB112348 | 1999 | Myanmar | AB205129 | 1999 | Ghana | D23681 | 1992 | Japan |
| AB112408 | 1999 | Myanmar | AB205188 | 2000 | Ghana | D23682 | 1984 | Japan |
| AB112471 | 2002 | Thailand | AB205189 | 2000 | Ghana | D23683 | 1984 | Japan |
| AB112472 | 2002 | Thailand | AB205190 | 2000 | Ghana | D23684 | 1988 | Japan |
| AB115417 | 1996 | Japan | AB205191 | 2000 | Ghana | D28880 | 1992 | Japan |
| AB115551 | 2004 | Cambodia | AB205192 | 2000 | Ghana | DQ060824 | 1983 | Namibia |
| AB116266 | 1987 | Japan | AB212625 | 2004 | Vietnam | DQ060825 | 1983 | Namibia |
| AB117758 | 2004 | Cambodia | AB212626 | 2004 | Vietnam | DQ060826 | 1983 | Namibia |
| AB117759 | 2004 | Cambodia | AB219426 | 2002 | Philippines | DQ060827 | 1983 | Namibia |
| AB119251 | 2002 | Japan | AB219427 | 2002 | Philippines | DQ060828 | 1983 | Namibia |
| AB119252 | 2002 | Japan | AB219428 | 2002 | Philippine | DQ060829 | 1983 | Namibia |
| AB119253 | 2000 | Japan | AB219429 | 2002 | Philippines | DQ315776 | 2000 | India |
| AB119254 | 2003 | Japan | AF121243 | 1995 | Vietnam | DQ315777 | 2003 | India |
| AB119255 | 2001 | Japan | AF121244 | 1993 | Vietnam | DQ315778 | 2003 | India |
| AB119256 | 1997 | Japan | AF121245 | 1992 | Vietnam | DQ315779 | 2003 | India |
| AB120308 | 1982 | Japan | AF121246 | 1998 | Vietnam | DQ315780 | 2002 | India |
| AB126580 | 2000 | Russia | AF121247 | 1994 | Vietnam | DQ315781 | 2003 | India |
| AB126581 | 2000 | Russia | AF121248 | 1994 | Vietnam | DQ315782 | 2004 | India |
| AB179747 | 2002 | Italy | AF121249 | 1998 | Vietnam | DQ315783 | 2003 | India |
| AB188241 | 1999 | Japan | AF121250 | 1997 | Vietnam | DQ315784 | 2003 | India |
| AB188242 | 1999 | Japan | AF121251 | 1997 | Vietnam | DQ315785 | 2003 | India |
| AB188243 | 1999 | Japan | AF182802 | 1992 | China | DQ315786 | 2005 | India |
| AB188245 | 2000 | Japan | AJ344117 | 1990 | France | M57663 | 1987 | Philippines |