Appendix A

Vigotsky *et al.*

Here, we show that the partial η^2 calculated using type III sums of squares (SS) is equivalent to how the variance accounted for (VAF) is derived for our hierarchical linear model (HLM). Notation is as follows: SSR = SS due to the regression; SSE = SS due to error (residual); SST = SS total ($SSR + SSE$); s^2 = explained absolute sample variance; s^2_{residual} = unexplained absolute sample variance.

Proof. Partial η^2 is equivalent to HLM VAF.

$$1 - \frac{s^2}{s^2_{\text{uncond}}} = \frac{SS_{size}}{SS_{size} + SS_{\text{residual}}} \tag{1}$$

$$= \frac{SSR\,\{\text{size} \mid \text{subject}\}}{SSR\,\{\text{size} \mid \text{subject}\} + SSE\,\{\text{size} \mid \text{subject}\}} \tag{2}$$

$$= \frac{\frac{1}{n-1} SSR\,\{\text{size} \mid \text{subject}\}}{\frac{1}{n-1} SSR\,\{\text{size} \mid \text{subject}\} + \frac{1}{n-1} SSE\,\{\text{size} \mid \text{subject}\}} \tag{3}$$

$$= \frac{s^2\,\{\text{size} \mid \text{subject}\}}{s^2\,\{\text{size} \mid \text{subject}\} + s^2_{\text{residual}}\,\{\text{size} \mid \text{subject}\}} \tag{4}$$

Following the $SST = SSR + SSE$ relationship, we know that $s^2_{\text{residual}}\,\{\text{subject}\} = s^2\,\{\text{size} \mid \text{subject}\} + s^2_{\text{residual}}\,\{\text{size} \mid \text{subject}\}$, so

$$= \frac{s^2\,\{\text{size} \mid \text{subject}\}}{s^2_{\text{residual}}\,\{\text{subject}\}} \tag{5}$$

$$= \frac{s^2_{\text{residual}}\,\{\text{subject}\} - s^2_{\text{residual}}\,\{\text{size} \mid \text{subject}\}}{s^2_{\text{residual}}\,\{\text{subject}\}} \tag{6}$$

$$= 1 - \frac{s^2_{\text{residual}}\,\{\text{size} \mid \text{subject}\}}{s^2_{\text{residual}}\,\{\text{subject}\}} \tag{7}$$

Finally, the sample variance of the residuals from the HLM were defined such that $s^2 = s^2_{\text{residual}}\,\{\text{size} \mid \text{subject}\}$ and $s^2_{\text{uncond}} = s^2_{\text{residual}}\,\{\text{subject}\}$; therefore,

$$= 1 - \frac{s^2}{s^2_{\text{uncond}}} \qquad \square$$