**Supplemental Information**

*Supplemental Article S1:**Detailed derivation on the estimation bias magnitude of Rao’s quadratic diversity index that is widely used in the ecological literature*

. (S1)

Recognizing that

. (S2)

Moreover, because it is well known that species abundance data are assumed to follow a multinomial distribution with rates  (Chao, 1981; Chao & Bunge, 2002; Shen, Chao & Lin, 2003; Chao & Jost, 2012; Chen & Shen, 2017; Shen, Chen & Chen, 2017), we have the following quantities (Basharin, 1959):

. (S3)

Therefore, by taking expectations on both sides of Eq. 2 and using the facts in Eq. S3, we get:

, (S4)

which is Eq. 3 in the main text.

*Supplemental Article S2:**A simple derivation of the unbiased Rao’s quadratic diversity index*

We thank Dr. Zoltán Botta-Dukát for providing this simple but heuristic proof. Recalling that Rao's quadratic diversity index can be interpreted as mean distance between a pair of individuals randomly chosen from the sample, therefore, it can be calculated as,

,



where  represents the probability that the first individual of the pair belongs to species *i* while the second one belongs to species *j*. Because ,  can be regarded as a weighted mean of all elements of the distance matrix, including zeros in the diagonal.

As mentioned above, since the first individual is expected to belong to species *i* while the second individual is expected to belong to another species *j*, it is easy to know that if , the unbiased estimate of  is,

.

By substituting this back to the previous formula, we get the unbiased estimator  for Rao's quadratic diversity index as shown in Eq. 6 of the main text.

*Supplemental Article S3:**Relationship between the unbiased Rao’s quadratic diversity index and the unbiased Gini-Simpson index*

Note that, given , ; , otherwise, the unbiased Rao’s quadratic diversity index can include the unbiased Gini-Simpson index (*Sim*) as a special case from the derivation below:

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