

Wealth and adjustment of social behaviour influence patterns of inequality and productivity across animal societies

Mathematica Code

Antonio M. M. Rodrigues

Department of Zoology, University of Cambridge, Cambridge CB2 3EJ, United Kingdom

Ecology

In[9872]:= **Clear**[α , p]

Eigensystem $\left[\begin{pmatrix} \text{pRR} & 1 - \text{pPP} \\ 1 - \text{pRR} & \text{pPP} \end{pmatrix}\right]$

Solve $\left[\left\{p == \frac{1 - \text{pRR}}{2 - (\text{pRR} + \text{pPP})}, \tau == \text{pRR} - (1 - \text{pPP})\right\}, \{\text{pRR}, \text{pPP}\}\right]$

Out[9873]= $\left\{\{1, -1 + \text{pPP} + \text{pRR}\}, \left\{\left\{-\frac{1 - \text{pPP}}{-1 + \text{pRR}}, 1\right\}, \{-1, 1\}\right\}\right\}$

Out[9874]= $\{\{\text{pRR} \rightarrow 1 - p + p \tau, \text{pPP} \rightarrow p + \tau - p \tau\}\}$

Relatedness

In[5118]:= **Clear**[fLR, fLP]

Solve[

$\left\{rR == (1 - p + p \tau) \varphi R \left(\left(\frac{fHR}{fHR + fLR} \right)^2 + \left(\frac{fLR}{fHR + fLR} \right)^2 + 2 \frac{fHR}{fHR + fLR} \frac{fLR}{fHR + fLR} rR \right) + \right.$

$(1 - (1 - p + p \tau)) \varphi P$

$\left(\left(\frac{fHP}{fHP + fLP} \right)^2 + \left(\frac{fLP}{fHP + fLP} \right)^2 + 2 \frac{fHP}{fHP + fLP} \frac{fLP}{fHP + fLP} rP \right),$

$rP == (p + \tau - p \tau) \varphi P \left(\left(\frac{fHP}{fHP + fLP} \right)^2 + \left(\frac{fLP}{fHP + fLP} \right)^2 + 2 \frac{fHP}{fHP + fLP} \frac{fLP}{fHP + fLP} rP \right) +$

$(1 - (p + \tau - p \tau)) \varphi R$

$\left(\left(\frac{fHR}{fHR + fLR} \right)^2 + \left(\frac{fLR}{fHR + fLR} \right)^2 + 2 \frac{fHR}{fHR + fLR} \frac{fLR}{fHR + fLR} rR \right)\},$

$\{rR, rP\}$]

$$\begin{aligned}
\text{Out}[5119] = & \left\{ \left\{ rR \rightarrow - \left(\left(1 - \frac{2 \text{fHP} \text{fLP} (p + \tau - p \tau) \varphi P}{(\text{fHP} + \text{fLP})^2} \right) \left(- \frac{\text{fHP}^2 (p - p \tau) \varphi P}{(\text{fHP} + \text{fLP})^2} - \frac{\text{fLP}^2 (p - p \tau) \varphi P}{(\text{fHP} + \text{fLP})^2} - \right. \right. \right. \\
& \left. \left. \frac{\text{fHR}^2 (1 - p + p \tau) \varphi R}{(\text{fHR} + \text{fLR})^2} - \frac{\text{fLR}^2 (1 - p + p \tau) \varphi R}{(\text{fHR} + \text{fLR})^2} \right) - \frac{1}{(\text{fHP} + \text{fLP})^2} \right. \\
& \left. 2 \text{fHP} \text{fLP} (p - p \tau) \varphi P \left(- \frac{\text{fHP}^2 (p + \tau - p \tau) \varphi P}{(\text{fHP} + \text{fLP})^2} - \frac{\text{fLP}^2 (p + \tau - p \tau) \varphi P}{(\text{fHP} + \text{fLP})^2} - \right. \right. \\
& \left. \left. \frac{\text{fHR}^2 (1 - p - \tau + p \tau) \varphi R}{(\text{fHR} + \text{fLR})^2} - \frac{\text{fLR}^2 (1 - p - \tau + p \tau) \varphi R}{(\text{fHR} + \text{fLR})^2} \right) \right) / \\
& \left(\frac{4 \text{fHP} \text{fHR} \text{fLP} \text{fLR} (p - p \tau) (1 - p - \tau + p \tau) \varphi P \varphi R}{(\text{fHP} + \text{fLP})^2 (\text{fHR} + \text{fLR})^2} - \right. \\
& \left. \left(1 - \frac{2 \text{fHP} \text{fLP} (p + \tau - p \tau) \varphi P}{(\text{fHP} + \text{fLP})^2} \right) \left(1 - \frac{2 \text{fHR} \text{fLR} (1 - p + p \tau) \varphi R}{(\text{fHR} + \text{fLR})^2} \right) \right) \right\}, \\
rP \rightarrow & - \left((\text{fHP}^2 \text{fHR}^2 p \varphi P + \text{fHR}^2 \text{fLP}^2 p \varphi P + 2 \text{fHP}^2 \text{fHR} \text{fLR} p \varphi P + 2 \text{fHR} \text{fLP}^2 \text{fLR} p \varphi P + \right. \\
& \text{fHP}^2 \text{fLR}^2 p \varphi P + \text{fLP}^2 \text{fLR}^2 p \varphi P + \text{fHP}^2 \text{fHR}^2 \tau \varphi P + \text{fHR}^2 \text{fLP}^2 \tau \varphi P + \\
& 2 \text{fHP}^2 \text{fHR} \text{fLR} \tau \varphi P + 2 \text{fHR} \text{fLP}^2 \text{fLR} \tau \varphi P + \text{fHP}^2 \text{fLR}^2 \tau \varphi P + \text{fLP}^2 \text{fLR}^2 \tau \varphi P - \\
& \text{fHP}^2 \text{fHR}^2 p \tau \varphi P - \text{fHR}^2 \text{fLP}^2 p \tau \varphi P - 2 \text{fHP}^2 \text{fHR} \text{fLR} p \tau \varphi P - 2 \text{fHR} \text{fLP}^2 \text{fLR} p \tau \varphi P - \\
& \text{fHP}^2 \text{fLR}^2 p \tau \varphi P - \text{fLP}^2 \text{fLR}^2 p \tau \varphi P + \text{fHP}^2 \text{fHR}^2 \varphi R + 2 \text{fHP} \text{fHR}^2 \text{fLP} \varphi R + \\
& \text{fHR}^2 \text{fLP}^2 \varphi R + \text{fHP}^2 \text{fLR}^2 \varphi R + 2 \text{fHP} \text{fLP} \text{fLR}^2 \varphi R + \text{fLP}^2 \text{fLR}^2 \varphi R - \text{fHP}^2 \text{fHR}^2 p \varphi R - \\
& 2 \text{fHP} \text{fHR}^2 \text{fLP} p \varphi R - \text{fHR}^2 \text{fLP}^2 p \varphi R - \text{fHP}^2 \text{fLR}^2 p \varphi R - 2 \text{fHP} \text{fLP} \text{fLR}^2 p \varphi R - \\
& \text{fLP}^2 \text{fLR}^2 p \varphi R - \text{fHP}^2 \text{fHR}^2 \tau \varphi R - 2 \text{fHP} \text{fHR}^2 \text{fLP} \tau \varphi R - \text{fHR}^2 \text{fLP}^2 \tau \varphi R - \\
& \text{fHP}^2 \text{fLR}^2 \tau \varphi R - 2 \text{fHP} \text{fLP} \text{fLR}^2 \tau \varphi R - \text{fLP}^2 \text{fLR}^2 \tau \varphi R + \text{fHP}^2 \text{fHR}^2 p \tau \varphi R + \\
& 2 \text{fHP} \text{fHR}^2 \text{fLP} p \tau \varphi R + \text{fHR}^2 \text{fLP}^2 p \tau \varphi R + \text{fHP}^2 \text{fLR}^2 p \tau \varphi R + 2 \text{fHP} \text{fLP} \text{fLR}^2 p \tau \varphi R + \\
& \left. \text{fLP}^2 \text{fLR}^2 p \tau \varphi R - 2 \text{fHP}^2 \text{fHR} \text{fLR} \tau \varphi P \varphi R - 2 \text{fHR} \text{fLP}^2 \text{fLR} \tau \varphi P \varphi R) / \right. \\
& \left(- \text{fHP}^2 \text{fHR}^2 - 2 \text{fHP} \text{fHR}^2 \text{fLP} - \text{fHR}^2 \text{fLP}^2 - 2 \text{fHP}^2 \text{fHR} \text{fLR} - 4 \text{fHP} \text{fHR} \text{fLP} \text{fLR} - \right. \\
& 2 \text{fHR} \text{fLP}^2 \text{fLR} - \text{fHP}^2 \text{fLR}^2 - 2 \text{fHP} \text{fLP} \text{fLR}^2 - \text{fLP}^2 \text{fLR}^2 + 2 \text{fHP} \text{fHR}^2 \text{fLP} p \varphi P + \\
& 4 \text{fHP} \text{fHR} \text{fLP} \text{fLR} p \varphi P + 2 \text{fHP} \text{fLP} \text{fLR}^2 p \varphi P + 2 \text{fHP} \text{fHR}^2 \text{fLP} \tau \varphi P + \\
& 4 \text{fHP} \text{fHR} \text{fLP} \text{fLR} \tau \varphi P + 2 \text{fHP} \text{fLP} \text{fLR}^2 \tau \varphi P - 2 \text{fHP} \text{fHR}^2 \text{fLP} p \tau \varphi P - \\
& 4 \text{fHP} \text{fHR} \text{fLP} \text{fLR} p \tau \varphi P - 2 \text{fHP} \text{fLP} \text{fLR}^2 p \tau \varphi P + 2 \text{fHP}^2 \text{fHR} \text{fLR} \varphi R + \\
& 4 \text{fHP} \text{fHR} \text{fLP} \text{fLR} \varphi R + 2 \text{fHR} \text{fLP}^2 \text{fLR} \varphi R - 2 \text{fHP}^2 \text{fHR} \text{fLR} p \varphi R - \\
& 4 \text{fHP} \text{fHR} \text{fLP} \text{fLR} p \varphi R - 2 \text{fHR} \text{fLP}^2 \text{fLR} p \varphi R + 2 \text{fHP}^2 \text{fHR} \text{fLR} p \tau \varphi R + \\
& \left. 4 \text{fHP} \text{fHR} \text{fLP} \text{fLR} p \tau \varphi R + 2 \text{fHR} \text{fLP}^2 \text{fLR} p \tau \varphi R - 4 \text{fHP} \text{fHR} \text{fLP} \text{fLR} \tau \varphi P \varphi R) \right) \} \}
\end{aligned}$$

```
Clear[d, rR, rP]
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```
c = 0.9;
```

```
d = 0.1;
```

```
μR = 1;
```

```
μP = 0.1;
```

```
qHR = 1;
```

```
qLR = 0.1;
```

```
qHP = 1;
```

```
qLP = 0.1;
```

```
hlabel = {{-1, "-1.0", {0, 0.01}}, {-0.5, "-0.5", {0, 0.01}},
           {0, "0.0", {0, 0.01}}, {0.5, "0.5", {0, 0.01}}},
```

```

{1.0, "1.0", {0, 0.01}}};
vlabel = Join[{{0.0, "0.0", {0, 0.01}}},
Table[{i, ToString[N[i]], {0, 0.01}}, {i,  $\frac{2}{10}$ , .8,  $\frac{2}{10}$ }],
{{1, "1.0", {0, 0.01}}}]];
pticks = Join[Table[{i, ToString[i], {0, 0.01}}, {i, -0.8, -0.2, 0.2}],
{{0, "0.0", {0, 0.01}}},
Table[{i, ToString[i], {0, 0.01}}, {i, 0.2, 0.8, 0.2}]];
colour = {"#DA3232", "#12588E"}, {"#DA3232", "#12588E"};
style1 = {Directive[RGBColor[colour[[1]][[1]]],
Directive[RGBColor[colour[[1]][[2]]], Dashing[{0.03, 0.05}],
Thickness[0.015]}};

```

$$f_{HR} := \mu_R \left(q_{HR} + \frac{x_{HR}}{\frac{1}{2} (x_{HR} + x_{LR})} \left(1 - \frac{1}{2} (x_{HR} + x_{LR}) \right) \right);$$

$$f_{LR} := \mu_R \left(q_{LR} + \frac{x_{LR}}{\frac{1}{2} (x_{HR} + x_{LR})} \left(1 - \frac{1}{2} (x_{HR} + x_{LR}) \right) \right);$$

$$f_{HP} := \mu_P \left(q_{HP} + \frac{x_{HP}}{\frac{1}{2} (x_{HP} + x_{LP})} \left(1 - \frac{1}{2} (x_{HP} + x_{LP}) \right) \right);$$

$$f_{LP} := \mu_P \left(q_{LP} + \frac{x_{LP}}{\frac{1}{2} (x_{HP} + x_{LP})} \left(1 - \frac{1}{2} (x_{HP} + x_{LP}) \right) \right);$$

$$s_H = \frac{f_{LR}}{f_{HR}}; s_L = \frac{f_{LP}}{f_{HP}}; \sigma = \frac{f_{HP}}{f_{HR}};$$

$$\varphi_R := \frac{((1-d)(f_{HR} + f_{LR}))}{((1-d)(f_{HR} + f_{LR}) + (p(f_{HR} + f_{LR}) + (1-p)(f_{HP} + f_{LP}))d(1-c))^2};$$

$$\varphi_P := \frac{((1-d)(f_{HP} + f_{LP}))}{((1-d)(f_{HP} + f_{LP}) + (p(f_{HR} + f_{LR}) + (1-p)(f_{HP} + f_{LP}))d(1-c))^2};$$

$$r_R[\tau_-] := - \left(\left(1 - \frac{2 f_{HP} f_{LP} (p + \tau - p \tau) \varphi_P}{(f_{HP} + f_{LP})^2} \right) \right)$$

$$\begin{aligned}
& \left(-\frac{fHP^2 (p - p \tau) \varphi P}{(fHP + fLP)^2} - \frac{fLP^2 (p - p \tau) \varphi P}{(fHP + fLP)^2} - \frac{fHR^2 (1 - p + p \tau) \varphi R}{(fHR + fLR)^2} - \right. \\
& \quad \left. \frac{fLR^2 (1 - p + p \tau) \varphi R}{(fHR + fLR)^2} \right) - \\
& \left(2 fHP fLP (p - p \tau) \varphi P \right. \\
& \quad \left(-\frac{fHP^2 (p + \tau - p \tau) \varphi P}{(fHP + fLP)^2} - \frac{fLP^2 (p + \tau - p \tau) \varphi P}{(fHP + fLP)^2} - \frac{fHR^2 (1 - p - \tau + p \tau) \varphi R}{(fHR + fLR)^2} - \right. \\
& \quad \left. \frac{fLR^2 (1 - p - \tau + p \tau) \varphi R}{(fHR + fLR)^2} \right) \Bigg) / (fHP + fLP)^2 \Bigg) / \\
& \left(\frac{4 fHP fHR fLP fLR (p - p \tau) (1 - p - \tau + p \tau) \varphi P \varphi R}{(fHP + fLP)^2 (fHR + fLR)^2} - \right. \\
& \quad \left. \left(1 - \frac{2 fHP fLP (p + \tau - p \tau) \varphi P}{(fHP + fLP)^2} \right) \left(1 - \frac{2 fHR fLR (1 - p + p \tau) \varphi R}{(fHR + fLR)^2} \right) \right) \Bigg)
\end{aligned}$$

rP[τ] :=

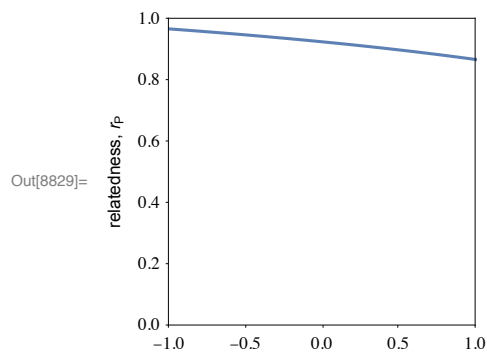
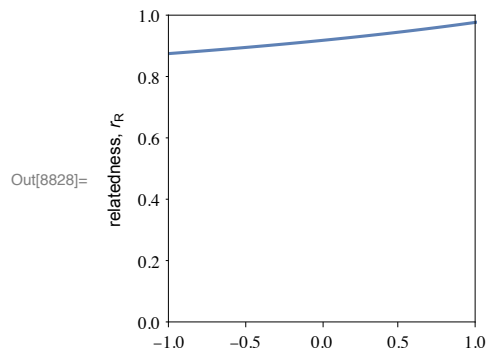
$$\begin{aligned}
& - \left((fHP^2 fHR^2 p \varphi P + fHR^2 fLP^2 p \varphi P + 2 fHP^2 fHR fLR p \varphi P + 2 fHR fLP^2 fLR p \varphi P + \right. \\
& \quad fHP^2 fLR^2 p \varphi P + fLP^2 fLR^2 p \varphi P + fHP^2 fHR^2 \tau \varphi P + fHR^2 fLP^2 \tau \varphi P + \\
& \quad 2 fHP^2 fHR fLR \tau \varphi P + 2 fHR fLP^2 fLR \tau \varphi P + fHP^2 fLR^2 \tau \varphi P + \\
& \quad fLP^2 fLR^2 \tau \varphi P - fHP^2 fHR^2 p \tau \varphi P - fHR^2 fLP^2 p \tau \varphi P - 2 fHP^2 fHR fLR p \tau \varphi P - \\
& \quad 2 fHR fLP^2 fLR p \tau \varphi P - fHP^2 fLR^2 p \tau \varphi P - fLP^2 fLR^2 p \tau \varphi P + \\
& \quad fHP^2 fHR^2 \varphi R + 2 fHP fHR^2 fLP \varphi R + fHR^2 fLP^2 \varphi R + fHP^2 fLR^2 \varphi R + \\
& \quad 2 fHP fLP fLR^2 \varphi R + fLP^2 fLR^2 \varphi R - fHP^2 fHR^2 p \varphi R - 2 fHP fHR^2 fLP p \varphi R - \\
& \quad fHR^2 fLP^2 p \varphi R - fHP^2 fLR^2 p \varphi R - 2 fHP fLP fLR^2 p \varphi R - fLP^2 fLR^2 p \varphi R - \\
& \quad fHP^2 fHR^2 \tau \varphi R - 2 fHP fHR^2 fLP \tau \varphi R - fHR^2 fLP^2 \tau \varphi R - fHP^2 fLR^2 \tau \varphi R - \\
& \quad 2 fHP fLP fLR^2 \tau \varphi R - fLP^2 fLR^2 \tau \varphi R + fHP^2 fHR^2 p \tau \varphi R + \\
& \quad 2 fHP fHR^2 fLP p \tau \varphi R + fHR^2 fLP^2 p \tau \varphi R + fHP^2 fLR^2 p \tau \varphi R + \\
& \quad 2 fHP fLP fLR^2 p \tau \varphi R + fLP^2 fLR^2 p \tau \varphi R - 2 fHP^2 fHR fLR \tau \varphi P \varphi R - \\
& \quad \left. 2 fHR fLP^2 fLR \tau \varphi P \varphi R) / \right. \\
& \quad (-fHP^2 fHR^2 - 2 fHP fHR^2 fLP - fHR^2 fLP^2 - 2 fHP^2 fHR fLR - \\
& \quad 4 fHP fHR fLP fLR - 2 fHR fLP^2 fLR - fHP^2 fLR^2 - 2 fHP fLP fLR^2 - \\
& \quad fLP^2 fLR^2 + 2 fHP fHR^2 fLP p \varphi P + 4 fHP fHR fLP fLR p \varphi P + \\
& \quad 2 fHP fLP fLR^2 p \varphi P + 2 fHP fHR^2 fLP \tau \varphi P + 4 fHP fHR fLP fLR \tau \varphi P + \\
& \quad 2 fHP fLP fLR^2 \tau \varphi P - 2 fHP fHR^2 fLP p \tau \varphi P - 4 fHP fHR fLP fLR p \tau \varphi P - \\
& \quad 2 fHP fLP fLR^2 p \tau \varphi P + 2 fHP^2 fHR fLR \varphi R + 4 fHP fHR fLP fLR \varphi R + \\
& \quad 2 fHR fLP^2 fLR \varphi R - 2 fHP^2 fHR fLR p \varphi R - 4 fHP fHR fLP fLR p \varphi R - \\
& \quad 2 fHR fLP^2 fLR p \varphi R + 2 fHP^2 fHR fLR p \tau \varphi R + 4 fHP fHR fLP fLR p \tau \varphi R + \\
& \quad \left. 2 fHR fLP^2 fLR p \tau \varphi R - 4 fHP fHR fLP fLR \tau \varphi P \varphi R) \right)
\end{aligned}$$

Plot[{rR[τ] /. {xHR → 1, xLR → 1, xHP → 1, xLP → 1}}, { τ , -1, 1},
PlotRange → {{-1, 1}, {0, 1}}, **Frame** → **True**, **AxesOrigin** → {-1, -0.0},

```

FrameLabel → {{Text[Style["relatedness,  $r_R$ "], None], {None, None}}},
FrameTicks → {{vlabel, None}, {hlabel, None}}, AspectRatio → 1,
ImageSize → 200]
Plot[{rP[ $\tau$ ] /. {xHR → 1, xLR → 1, xHP → 1, xLP → 1}}, { $\tau$ , -1, 1},
PlotRange → {{-1, 1}, {0, 1}}, Frame → True, AxesOrigin → {-1, -0.0},
FrameLabel → {{Text[Style["relatedness,  $r_P$ "], None], {None, None}}},
FrameTicks → {{vlabel, None}, {hlabel, None}}, AspectRatio → 1,
ImageSize → 200]

```



Reproductive value calculations

```

wHRHR[ $\tau_{-}$ ] :=
((1 - d) / ((1 + sR) (1 - d) + (p (1 + sR) + (1 - p)  $\sigma$  (1 + sP)) d (1 - c)))
(p +  $\tau$  - p  $\tau$ ) +
((d (1 - c) p) / ((1 + sR) (1 - d) + (p (1 + sR) + (1 - p)  $\sigma$  (1 + sP)) d (1 - c)))
(p +  $\tau$  - p  $\tau$ ) +
((d (1 - c) (1 - p)) /
( $\sigma$  (1 + sP) (1 - d) + (p (1 + sR) + (1 - p)  $\sigma$  (1 + sP)) d (1 - c)))
(1 - (1 - p + p  $\tau$ )) ;
wHRLR[ $\tau_{-}$ ] :=
((1 - d) / ((1 + sR) (1 - d) + (p (1 + sR) + (1 - p)  $\sigma$  (1 + sP)) d (1 - c)))
(p +  $\tau$  - p  $\tau$ ) +
((d (1 - c) p) / ((1 + sR) (1 - d) + (p (1 + sR) + (1 - p)  $\sigma$  (1 + sP)) d (1 - c)))
(p +  $\tau$  - p  $\tau$ ) +

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$$\frac{(d(1-c)(1-p))}{(\sigma(1+sP)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))} \\ (1 - (1-p+p\tau));$$

wHRHP $[\tau_]$:=

$$\frac{((1-d) / ((1+sR)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ (1 - (p+\tau-p\tau)) + \\ ((d(1-c)p) / ((1+sR)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ (1 - (p+\tau-p\tau)) + \\ ((d(1-c)(1-p)) / \\ (\sigma(1+sP)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) (1-p+p\tau);$$

wHRLP $[\tau_]$:=

$$\frac{((1-d) / ((1+sR)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ (1 - (p+\tau-p\tau)) + \\ ((d(1-c)p) / ((1+sR)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ (1 - (p+\tau-p\tau)) + \\ ((d(1-c)(1-p)) / \\ (\sigma(1+sP)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) (1-p+p\tau);$$

wHPHP $[\tau_]$:=

$$\frac{((\sigma(1-d)) / (\sigma(1+sP)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ (1-p+p\tau) + \\ ((\sigma d(1-c)p) / ((1+sR)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ (1 - (p+\tau-p\tau)) + \\ ((\sigma d(1-c)(1-p)) / \\ (\sigma(1+sP)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) (1-p+p\tau);$$

wHPLP $[\tau_]$:=

$$\frac{((\sigma(1-d)) / (\sigma(1+sP)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ (1-p+p\tau) + \\ ((\sigma d(1-c)p) / ((1+sR)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ (1 - (p+\tau-p\tau)) + \\ ((\sigma d(1-c)(1-p)) / \\ (\sigma(1+sP)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) (1-p+p\tau);$$

wHPHR $[\tau_]$:=

$$\frac{((\sigma(1-d)) / (\sigma(1+sP)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ (1 - (1-p+p\tau)) + \\ ((\sigma d(1-c)p) / ((1+sR)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ (p+\tau-p\tau) + \\ ((\sigma d(1-c)(1-p)) /$$

```

      (σ (1 + sP) (1 - d) + (p (1 + sR) + (1 - p) σ (1 + sP)) d (1 - c))
      (1 - (1 - p + p τ));
wHPLR[τ_] :=
  ((σ (1 - d)) / (σ (1 + sP) (1 - d) + (p (1 + sR) + (1 - p) σ (1 + sP)) d (1 - c)))
  (1 - (1 - p + p τ)) +
  ((σ d (1 - c) p) / ((1 + sR) (1 - d) + (p (1 + sR) + (1 - p) σ (1 + sP)) d (1 - c)))
  (p + τ - p τ) +
  ((σ d (1 - c) (1 - p)) /
    (σ (1 + sP) (1 - d) + (p (1 + sR) + (1 - p) σ (1 + sP)) d (1 - c)))
  (1 - (1 - p + p τ));

wLRLR[τ_] :=
  ((sR (1 - d)) / ((1 + sR) (1 - d) + (p (1 + sR) + (1 - p) σ (1 + sP)) d (1 - c)))
  (p + τ - p τ) +
  ((sR d (1 - c) p) / ((1 + sR) (1 - d) + (p (1 + sR) + (1 - p) σ (1 + sP)) d (1 - c)))
  (p + τ - p τ) +
  ((sR d (1 - c) (1 - p)) /
    (σ (1 + sP) (1 - d) + (p (1 + sR) + (1 - p) σ (1 + sP)) d (1 - c)))
  (1 - (1 - p + p τ));
wLRHR[τ_] :=
  ((sR (1 - d)) / ((1 + sR) (1 - d) + (p (1 + sR) + (1 - p) σ (1 + sP)) d (1 - c)))
  (p + τ - p τ) +
  ((sR d (1 - c) p) / ((1 + sR) (1 - d) + (p (1 + sR) + (1 - p) σ (1 + sP)) d (1 - c)))
  (p + τ - p τ) +
  ((sR d (1 - c) (1 - p)) /
    (σ (1 + sP) (1 - d) + (p (1 + sR) + (1 - p) σ (1 + sP)) d (1 - c)))
  (1 - (1 - p + p τ));

wLRLP[τ_] :=
  ((sR (1 - d)) / ((1 + sR) (1 - d) + (p (1 + sR) + (1 - p) σ (1 + sP)) d (1 - c)))
  (1 - (p + τ - p τ)) +
  ((sR d (1 - c) p) / ((1 + sR) (1 - d) + (p (1 + sR) + (1 - p) σ (1 + sP)) d (1 - c)))
  (1 - (p + τ - p τ)) +
  ((sR d (1 - c) (1 - p)) /
    (σ (1 + sP) (1 - d) + (p (1 + sR) + (1 - p) σ (1 + sP)) d (1 - c)))
  (1 - p + p τ);
wLRHP[τ_] :=
  ((sR (1 - d)) / ((1 + sR) (1 - d) + (p (1 + sR) + (1 - p) σ (1 + sP)) d (1 - c)))
  (1 - (p + τ - p τ)) +
  ((sR d (1 - c) p) / ((1 + sR) (1 - d) + (p (1 + sR) + (1 - p) σ (1 + sP)) d (1 - c)))
  (1 - (p + τ - p τ)) +
  ((sR d (1 - c) (1 - p)) /

```

$$(\sigma(1+sP)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c)) (1-p+p\tau);$$

wLPLP $[\tau_]$:=

$$\begin{aligned} & ((\sigma sP(1-d)) / (\sigma(1+sP)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ & (1-p+p\tau) + \\ & ((\sigma sP d(1-c)p) / \\ & ((1+sR)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ & (1-(p+\tau-p\tau)) + \\ & ((\sigma sP d(1-c)(1-p)) / \\ & (\sigma(1+sP)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) (1-p+p\tau); \end{aligned}$$

wLPHP $[\tau_]$:=

$$\begin{aligned} & ((\sigma sP(1-d)) / (\sigma(1+sP)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ & (1-p+p\tau) + \\ & ((\sigma sP d(1-c)p) / \\ & ((1+sR)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ & (1-(p+\tau-p\tau)) + \\ & ((\sigma sP d(1-c)(1-p)) / \\ & (\sigma(1+sP)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) (1-p+p\tau); \end{aligned}$$

wLPLR $[\tau_]$:=

$$\begin{aligned} & ((\sigma sP(1-d)) / (\sigma(1+sP)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ & (1-(1-p+p\tau)) + \\ & ((\sigma sP d(1-c)p) / \\ & ((1+sR)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) (p+\tau-p\tau) + \\ & ((\sigma sP d(1-c)(1-p)) / \\ & (\sigma(1+sP)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ & (1-(1-p+p\tau)); \end{aligned}$$

wLPHR $[\tau_]$:=

$$\begin{aligned} & ((\sigma sP(1-d)) / (\sigma(1+sP)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ & (1-(1-p+p\tau)) + \\ & ((\sigma sP d(1-c)p) / \\ & ((1+sR)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) (p+\tau-p\tau) + \\ & ((\sigma sP d(1-c)(1-p)) / \\ & (\sigma(1+sP)(1-d) + (p(1+sR) + (1-p)\sigma(1+sP))d(1-c))) \\ & (1-(1-p+p\tau)); \end{aligned}$$

$$A[\tau_]:= \begin{pmatrix} wHRHR[\tau] & wLRHR[\tau] & wHPHR[\tau] & wLPHR[\tau] \\ wHRLR[\tau] & wLRLR[\tau] & wHPLR[\tau] & wLPLR[\tau] \\ wHRHP[\tau] & wLRHP[\tau] & wHPHP[\tau] & wLPHP[\tau] \\ wHRLP[\tau] & wLRLP[\tau] & wHPLP[\tau] & wLPLP[\tau] \end{pmatrix};$$

AP = A[τ];

Eigenvalues[AP] // Simplify

Eigensystem[AP] // Simplify

Eigensystem[Transpose[AP]] // Simplify

Out[9578]= $\left\{0, 0, 1, \frac{4(-1+d)(-1+cd)(1+sP)(1+sR)\sigma\tau}{(-2(1+sR)+d(1+sR-\sigma-sP\sigma+c(1+sR+\sigma+sP\sigma))(-2(1+sP)\sigma+d(-1-sR+\sigma+sP\sigma+c(1+sR+\sigma+sP\sigma)))}\right\}$

Out[9579]= $\left\{\left\{0, 0, 1, \frac{4(-1+d)(-1+cd)(1+sP)(1+sR)\sigma\tau}{(-2(1+sR)+d(1+sR-\sigma-sP\sigma+c(1+sR+\sigma+sP\sigma))(-2(1+sP)\sigma+d(-1-sR+\sigma+sP\sigma+c(1+sR+\sigma+sP\sigma)))}\right\}, \left\{\{0, 0, -sP, 1\}, \{-sR, 1, 0, 0\}, \{1, 1, 1, 1\}, \left[-\left(\frac{(1+sP)\sigma(-2(1+sR)(-1+\tau)+d^2(1+sR-\sigma-sP\sigma+c^2(1+sR+\sigma+sP\sigma)-2c(1+sR)\tau)+2d(-1+\sigma+sP\sigma+sR(-1+\tau)+\tau-c(1+sR+\sigma+sP\sigma-\tau-sR\tau))}{(1+sR)(-2(1+sP)\sigma(-1+\tau)-2d(-1-sR+\sigma+sP\sigma+c(1+sR-(1+sP)\sigma(-1+\tau))-\sigma\tau-sP\sigma\tau)+d^2(-1-sR+\sigma+sP\sigma+c^2(1+sR+\sigma+sP\sigma)-2c(1+sP)\sigma\tau))}\right), -\left(\frac{(1+sP)\sigma(-2(1+sR)(-1+\tau)+d^2(1+sR-\sigma-sP\sigma+c^2(1+sR+\sigma+sP\sigma)-2c(1+sR)\tau)+2d(-1+\sigma+sP\sigma+sR(-1+\tau)+\tau-c(1+sR+\sigma+sP\sigma-\tau-sR\tau))}{(1+sR)(-2(1+sP)\sigma(-1+\tau)-2d(-1-sR+\sigma+sP\sigma+c(1+sR-(1+sP)\sigma(-1+\tau))-\sigma\tau-sP\sigma\tau)+d^2(-1-sR+\sigma+sP\sigma+c^2(1+sR+\sigma+sP\sigma)-2c(1+sP)\sigma\tau))}\right)\right\}, 1, 1\}\right\}$

Out[9580]= $\left\{\left\{0, 0, 1, \frac{4(-1+d)(-1+cd)(1+sP)(1+sR)\sigma\tau}{(-2(1+sR)+d(1+sR-\sigma-sP\sigma+c(1+sR+\sigma+sP\sigma))(-2(1+sP)\sigma+d(-1-sR+\sigma+sP\sigma+c(1+sR+\sigma+sP\sigma)))}\right\}, \left\{\{0, 0, -1, 1\}, \{-1, 1, 0, 0\}, \left\{-2(1+sP)\sigma(-1+\tau)-2d(-1-sR+\sigma+sP\sigma+c(1+sR-(1+sP)\sigma(-1+\tau))-\sigma\tau-sP\sigma\tau)+d^2(-1-sR+\sigma+sP\sigma+c^2(1+sR+\sigma+sP\sigma)-2c(1+sP)\sigma\tau)\right\}/(sP\sigma(-2(1+sR)(-1+\tau)+d^2(1+sR-\sigma-sP\sigma+c^2(1+sR+\sigma+sP\sigma)-2c(1+sR)\tau)+2d(-1+\sigma+sP\sigma+sR(-1+\tau)+\tau-c(1+sR+\sigma+sP\sigma-\tau-sR\tau))), (sR(-2(1+sP)\sigma(-1+\tau)-2d(-1-sR+\sigma+sP\sigma+c(1+sR-(1+sP)\sigma(-1+\tau))-\sigma\tau-sP\sigma\tau)+d^2(-1-sR+\sigma+sP\sigma+c^2(1+sR+\sigma+sP\sigma)-2c(1+sP)\sigma\tau)))/(sP\sigma(-2(1+sR)(-1+\tau)+d^2(1+sR-\sigma-sP\sigma+c^2(1+sR+\sigma+sP\sigma)-2c(1+sR)\tau)+2d(-1+\sigma+sP\sigma+sR(-1+\tau)+\tau-c(1+sR+\sigma+sP\sigma-\tau-sR\tau)))}, \frac{1}{sP}, 1\right\}, \left\{-\frac{1+sP}{sP+sP sR}, -\frac{sR+sP sR}{sP+sP sR}, \frac{1}{sP}, 1\right\}\right\}$

Plot

c = 0.9;

d = 0.1;

μR = 1;

μP = 0.1;

qHR = 1;

$$q_{LR} = 0.1;$$

$$q_{HP} = 1;$$

$$q_{LP} = 0.1;$$

$$f_{HR} := \mu_R \left(q_{HR} + \frac{x_{HR}}{\frac{1}{2} (x_{HR} + x_{LR})} \left(1 - \frac{1}{2} (x_{HR} + x_{LR}) \right) \right);$$

$$f_{LR} := \mu_R \left(q_{LR} + \frac{x_{LR}}{\frac{1}{2} (x_{HR} + x_{LR})} \left(1 - \frac{1}{2} (x_{HR} + x_{LR}) \right) \right);$$

$$f_{HP} := \mu_P \left(q_{HP} + \frac{x_{HP}}{\frac{1}{2} (x_{HP} + x_{LP})} \left(1 - \frac{1}{2} (x_{HP} + x_{LP}) \right) \right);$$

$$f_{LP} := \mu_P \left(q_{LP} + \frac{x_{LP}}{\frac{1}{2} (x_{HP} + x_{LP})} \left(1 - \frac{1}{2} (x_{HP} + x_{LP}) \right) \right);$$

$$s_H = \frac{f_{LR}}{f_{HR}}; s_L = \frac{f_{LP}}{f_{HP}}; \sigma = \frac{f_{HP}}{f_{HR}};$$

$$v_{HR}[\tau_] := 1$$

$$v_{LR}[\tau_] :=$$

$$\begin{aligned} & \left(s_H \right. \\ & \quad \left(-2 (1 + s_L) \sigma (-1 + \tau) - \right. \\ & \quad \quad 2 d (-1 - s_H + \sigma + s_L \sigma + c (1 + s_H - (1 + s_L) \sigma (-1 + \tau)) - \sigma \tau - s_L \sigma \tau) + \\ & \quad \quad \left. d^2 (-1 - s_H + \sigma + s_L \sigma + c^2 (1 + s_H + \sigma + s_L \sigma) - 2 c (1 + s_L) \sigma \tau) \right) \Big) / \\ & \left(s_L \sigma \right. \\ & \quad \left(-2 (1 + s_H) (-1 + \tau) + \right. \\ & \quad \quad d^2 (1 + s_H - \sigma - s_L \sigma + c^2 (1 + s_H + \sigma + s_L \sigma) - 2 c (1 + s_H) \tau) + \\ & \quad \quad \left. 2 d (-1 + \sigma + s_L \sigma + s_H (-1 + \tau) + \tau - c (1 + s_H + \sigma + s_L \sigma - \tau - s_H \tau)) \right) \Big) / \\ & \left(-2 (1 + s_L) \sigma (-1 + \tau) - \right. \\ & \quad 2 d (-1 - s_H + \sigma + s_L \sigma + c (1 + s_H - (1 + s_L) \sigma (-1 + \tau)) - \sigma \tau - s_L \sigma \tau) + \\ & \quad \left. d^2 (-1 - s_H + \sigma + s_L \sigma + c^2 (1 + s_H + \sigma + s_L \sigma) - 2 c (1 + s_L) \sigma \tau) \right) \Big) / \\ & \left(s_L \sigma \right. \\ & \quad \left(-2 (1 + s_H) (-1 + \tau) + \right. \\ & \quad \quad d^2 (1 + s_H - \sigma - s_L \sigma + c^2 (1 + s_H + \sigma + s_L \sigma) - 2 c (1 + s_H) \tau) + \\ & \quad \quad \left. 2 d (-1 + \sigma + s_L \sigma + s_H (-1 + \tau) + \tau - c (1 + s_H + \sigma + s_L \sigma - \tau - s_H \tau)) \right) \Big) \end{aligned}$$

$$v_{HP}[\tau_] :=$$

$$\begin{aligned} & \frac{1}{s_L} / \\ & \left(-2 (1 + s_L) \sigma (-1 + \tau) - \right. \\ & \quad \left. 2 d (-1 - s_H + \sigma + s_L \sigma + c (1 + s_H - (1 + s_L) \sigma (-1 + \tau)) - \sigma \tau - s_L \sigma \tau) + \right. \end{aligned}$$

```

      d2 (−1 − sH + σ + sL σ + c2 (1 + sH + σ + sL σ) − 2 c (1 + sL) σ τ) /
    (sL σ
      (−2 (1 + sH) (−1 + τ) +
        d2 (1 + sH − σ − sL σ + c2 (1 + sH + σ + sL σ) − 2 c (1 + sH) τ) +
        2 d (−1 + σ + sL σ + sH (−1 + τ) + τ − c (1 + sH + σ + sL σ − τ − sH τ)) ) )
vLP[τ_] :=
  1 /
  (−2 (1 + sL) σ (−1 + τ) −
    2 d (−1 − sH + σ + sL σ + c (1 + sH − (1 + sL) σ (−1 + τ)) − σ τ − sL σ τ) +
    d2 (−1 − sH + σ + sL σ + c2 (1 + sH + σ + sL σ) − 2 c (1 + sL) σ τ) /
  (sL σ
    (−2 (1 + sH) (−1 + τ) +
      d2 (1 + sH − σ − sL σ + c2 (1 + sH + σ + sL σ) − 2 c (1 + sH) τ) +
      2 d (−1 + σ + sL σ + sH (−1 + τ) + τ − c (1 + sH + σ + sL σ − τ − sH τ)) ) )

wHRφ[τ_] :=
  ((1 − d) (fHR)) /
  ((1 − d) (fHR + fLR) + (p (fHR + fLR) + (1 − p) (fHP + fLP)) d (1 − c))
wLRφ[τ_] :=
  ((1 − d) (fLR)) /
  ((1 − d) (fHR + fLR) + (p (fHR + fLR) + (1 − p) (fHP + fLP)) d (1 − c))
wHPφ[τ_] :=
  ((1 − d) (fHP)) /
  ((1 − d) (fHP + fLP) + (p (fHR + fLR) + (1 − p) (fHP + fLP)) d (1 − c))
wLPφ[τ_] :=
  ((1 − d) (fLP)) /
  ((1 − d) (fHP + fLP) + (p (fHR + fLR) + (1 − p) (fHP + fLP)) d (1 − c))

vHRφ[τ_] :=
  wHRφ[τ] ((1 − p + p τ) (vHR[τ] + vLR[τ]) + (1 − (1 − p + p τ)) (vHP[τ] + vLP[τ]))
vLRφ[τ_] :=
  wLRφ[τ] ((1 − p + p τ) (vHR[τ] + vLR[τ]) + (1 − (1 − p + p τ)) (vHP[τ] + vLP[τ]))
vHPφ[τ_] :=
  wHPφ[τ] ((1 − (p + τ − p τ)) (vHR[τ] + vLR[τ]) + (p + τ − p τ) (vHP[τ] + vLP[τ]))
vLPφ[τ_] :=
  wLPφ[τ] ((1 − (p + τ − p τ)) (vHR[τ] + vLR[τ]) + (p + τ − p τ) (vHP[τ] + vLP[τ]))

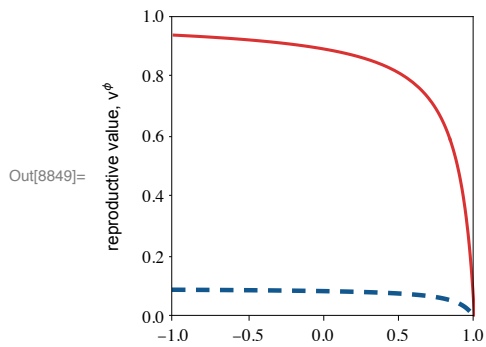
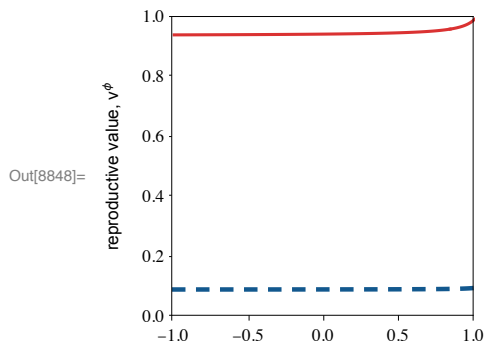
Plot[{vHRφ[τ] /. {xHR → 1, xLR → 1, xHP → 1, xLP → 1},
  vLRφ[τ] /. {xHR → 1, xLR → 1, xHP → 1, xLP → 1}}, {τ, −1, 1},
  PlotRange → {{−1, 1}, {0, 1}}, Frame → True, AxesOrigin → {−1, −0.0},
  FrameLabel → {{Text[Style["reproductive value, vφ"], None], None},

```

```

{None, None}}, FrameTicks → {{vlabel, None}, {hlabel, None}},
PlotStyle → style1, AspectRatio → 1, ImageSize → 200]
Plot[{vHP $\phi$ [ $\tau$ ] /. {xHR → 1, xLR → 1, xHP → 1, xLP → 1},
vLP $\phi$ [ $\tau$ ] /. {xHR → 1, xLR → 1, xHP → 1, xLP → 1}}, { $\tau$ , -1, 1},
PlotRange → {{-1, 1}, {0, 1}}, Frame → True, AxesOrigin → {-1, -0.0},
FrameLabel → {{Text[Style["reproductive value,  $v^\phi$ "]], None},
{None, None}}, FrameTicks → {{vlabel, None}, {hlabel, None}},
PlotStyle → style1, AspectRatio → 1, ImageSize → 200]

```



Potential for helping

$$\omega R[\tau_-] := \frac{(1-d)}{(1-d)(fHR+fLR) + (p(fHR+fLR) + (1-p)(fHP+fLP))d(1-c)}$$

$$\omega P[\tau_-] := \frac{(1-d)}{(1-d)(fHP+fLP) + (p(fHR+fLR) + (1-p)(fHP+fLP))d(1-c)}$$

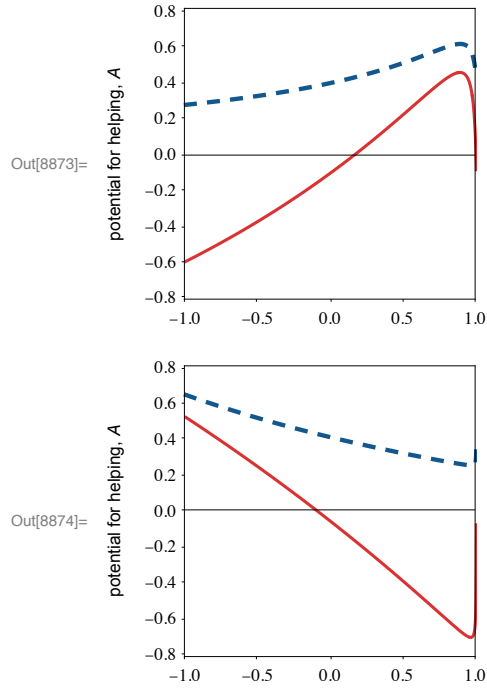
$$AHR[\tau_-] := \frac{\frac{vLR[\tau]}{fLR} rR[\tau] - \omega R[\tau] (vHR\phi[\tau] + vLR\phi[\tau] rR[\tau])}{\frac{vHR[\tau]}{fHR} - \omega R[\tau] (vHR\phi[\tau] + vLR\phi[\tau] rR[\tau])};$$

$$ALR[\tau_-] := \frac{\frac{vHR[\tau]}{fHR} rR[\tau] - \omega R[\tau] (vHR\phi[\tau] rR[\tau] + vLR\phi[\tau])}{\frac{vLR[\tau]}{fLR} - \omega R[\tau] (vHR\phi[\tau] rR[\tau] + vLR\phi[\tau])};$$

$$AHP[\tau_-] := \frac{\frac{vLP[\tau]}{fLP} rP[\tau] - \omega P[\tau] (vHP\phi[\tau] + vLP\phi[\tau] rP[\tau])}{\frac{vHP[\tau]}{fHP} - \omega P[\tau] (vHP\phi[\tau] + vLP\phi[\tau] rP[\tau])};$$

$$ALP[\tau_-] := \frac{\frac{vHP[\tau]}{fHP} rP[\tau] - \omega P[\tau] (vHP\phi[\tau] rP[\tau] + vLP\phi[\tau])}{\frac{vLP[\tau]}{fLP} - \omega P[\tau] (vHP\phi[\tau] rP[\tau] + vLP\phi[\tau])};$$

```
Plot[{AHR[τ] /. {xHR → 1, xLR → 1, xHP → 1, xLP → 1},
  ALR[τ] /. {xHR → 1, xLR → 1, xHP → 1, xLP → 1}}, {τ, -1, 1},
PlotRange → {{-1, 1}, {-0.81, 0.81}}, Frame → True,
FrameLabel → {{Text[Style["potential for helping, A"]], None},
  {None, None}}, AxesOrigin → {-1, -0.0},
FrameTicks → {{pticks, None}, {hlabel, None}}, PlotStyle → style1,
AspectRatio → 1, ImageSize → 200]
Plot[{AHP[τ] /. {xHR → 1, xLR → 1, xHP → 1, xLP → 1},
  ALP[τ] /. {xHR → 1, xLR → 1, xHP → 1, xLP → 1}}, {τ, -1, 1},
PlotRange → {{-1, 1}, {-0.81, 0.81}}, Frame → True,
FrameTicks → {{pticks, None}, {hlabel, None}},
FrameLabel → {{Text[Style["potential for helping, A"]], None},
  {None, None}}, AxesOrigin → {-1, -0.0}, PlotStyle → style1,
AspectRatio → 1, ImageSize → 200]
```



Optimal behaviour

c = 0.9;
d = 0.1;
 $\mu_R = 1$;
 $\mu_P = 0.1$;
qHR = 1;
qLR = 0.1;
qHP = 1;
qLP = 0.1;

$$f_{HR} := \mu_R \left(q_{HR} + \frac{x_{HR}}{\frac{1}{2}(x_{HR} + x_{LR})} \left(1 - \frac{1}{2}(x_{HR} + x_{LR}) \right) \right);$$

$$f_{LR} := \mu_R \left(q_{LR} + \frac{x_{LR}}{\frac{1}{2}(x_{HR} + x_{LR})} \left(1 - \frac{1}{2}(x_{HR} + x_{LR}) \right) \right);$$

$$f_{HP} := \mu_P \left(q_{HP} + \frac{x_{HP}}{\frac{1}{2}(x_{HP} + x_{LP})} \left(1 - \frac{1}{2}(x_{HP} + x_{LP}) \right) \right);$$

$$f_{LP} := \mu_P \left(q_{LP} + \frac{x_{LP}}{\frac{1}{2}(x_{HP} + x_{LP})} \left(1 - \frac{1}{2}(x_{HP} + x_{LP}) \right) \right);$$

$$s_R = \frac{f_{LR}}{f_{HR}}; \quad s_P = \frac{f_{LP}}{f_{HP}}; \quad \sigma = \frac{f_{HP}}{f_{HR}};$$

$$cHR := -\frac{1}{\frac{1}{2}} \left(1 - \frac{1}{2} (xHR + xLR) \right);$$

$$bHR := -\frac{xHR}{\frac{1}{2} (xHR + xLR)};$$

$$cLR := -\frac{1}{\frac{1}{2}} \left(1 - \frac{1}{2} (xHR + xLR) \right);$$

$$bLR := -\frac{xLR}{\frac{1}{2} (xHR + xLR)};$$

$$cHP := -\frac{1}{\frac{1}{2}} \left(1 - \frac{1}{2} (xHP + xLP) \right);$$

$$bHP := -\frac{xHP}{\frac{1}{2} (xHP + xLP)};$$

$$cLP := -\frac{1}{\frac{1}{2}} \left(1 - \frac{1}{2} (xHP + xLP) \right);$$

$$bLP := -\frac{xLP}{\frac{1}{2} (xHP + xLP)};$$

$$vHR[\tau_] := 1$$

$$vLR[\tau_] :=$$

$$\begin{aligned} & \left(sR \right. \\ & \quad \left(-2 (1 + sP) \sigma (-1 + \tau) - \right. \\ & \quad \quad 2 d (-1 - sR + \sigma + sP \sigma + c (1 + sR - (1 + sP) \sigma (-1 + \tau)) - \sigma \tau - sP \sigma \tau) + \\ & \quad \quad \left. d^2 (-1 - sR + \sigma + sP \sigma + c^2 (1 + sR + \sigma + sP \sigma) - 2 c (1 + sP) \sigma \tau) \right) \left. \right) / \\ & \left(sP \sigma \right. \\ & \quad \left(-2 (1 + sR) (-1 + \tau) + \right. \\ & \quad \quad d^2 (1 + sR - \sigma - sP \sigma + c^2 (1 + sR + \sigma + sP \sigma) - 2 c (1 + sR) \tau) + \\ & \quad \quad \left. 2 d (-1 + \sigma + sP \sigma + sR (-1 + \tau) + \tau - c (1 + sR + \sigma + sP \sigma - \tau - sR \tau) \right) \left. \right) \left. \right) / \\ & \left(-2 (1 + sP) \sigma (-1 + \tau) - \right. \\ & \quad 2 d (-1 - sR + \sigma + sP \sigma + c (1 + sR - (1 + sP) \sigma (-1 + \tau)) - \sigma \tau - sP \sigma \tau) + \\ & \quad \left. d^2 (-1 - sR + \sigma + sP \sigma + c^2 (1 + sR + \sigma + sP \sigma) - 2 c (1 + sP) \sigma \tau) \right) / \\ & \left(sP \sigma \right. \\ & \quad \left(-2 (1 + sR) (-1 + \tau) + \right. \end{aligned}$$

$$\begin{aligned}
& d^2 \left((1 + sR - \sigma - sP \sigma + c^2 (1 + sR + \sigma + sP \sigma) - 2 c (1 + sR) \tau) + \right. \\
& \left. 2 d (-1 + \sigma + sP \sigma + sR (-1 + \tau) + \tau - c (1 + sR + \sigma + sP \sigma - \tau - sR \tau)) \right) \\
\text{vHP}[\tau_]&:= \\
& \frac{1}{sP} / \\
& \left(-2 (1 + sP) \sigma (-1 + \tau) - \right. \\
& \left. 2 d (-1 - sR + \sigma + sP \sigma + c (1 + sR - (1 + sP) \sigma (-1 + \tau)) - \sigma \tau - sP \sigma \tau) + \right. \\
& \left. d^2 (-1 - sR + \sigma + sP \sigma + c^2 (1 + sR + \sigma + sP \sigma) - 2 c (1 + sP) \sigma \tau) \right) / \\
& \left(sP \sigma \right. \\
& \left. (-2 (1 + sR) (-1 + \tau) + \right. \\
& \left. d^2 (1 + sR - \sigma - sP \sigma + c^2 (1 + sR + \sigma + sP \sigma) - 2 c (1 + sR) \tau) + \right. \\
& \left. 2 d (-1 + \sigma + sP \sigma + sR (-1 + \tau) + \tau - c (1 + sR + \sigma + sP \sigma - \tau - sR \tau)) \right) \\
\text{vLP}[\tau_]&:= \\
& 1 / \\
& \left(-2 (1 + sP) \sigma (-1 + \tau) - \right. \\
& \left. 2 d (-1 - sR + \sigma + sP \sigma + c (1 + sR - (1 + sP) \sigma (-1 + \tau)) - \sigma \tau - sP \sigma \tau) + \right. \\
& \left. d^2 (-1 - sR + \sigma + sP \sigma + c^2 (1 + sR + \sigma + sP \sigma) - 2 c (1 + sP) \sigma \tau) \right) / \\
& \left(sP \sigma \right. \\
& \left. (-2 (1 + sR) (-1 + \tau) + \right. \\
& \left. d^2 (1 + sR - \sigma - sP \sigma + c^2 (1 + sR + \sigma + sP \sigma) - 2 c (1 + sR) \tau) + \right. \\
& \left. 2 d (-1 + \sigma + sP \sigma + sR (-1 + \tau) + \tau - c (1 + sR + \sigma + sP \sigma - \tau - sR \tau)) \right) \\
\text{wHR}\phi[\tau_]&:= \\
& ((1 - d) (fHR)) / \\
& ((1 - d) (fHR + fLR) + (p (fHR + fLR) + (1 - p) (fHP + fLP)) d (1 - c)) \\
\text{wLR}\phi[\tau_]&:= \\
& ((1 - d) (fLR)) / \\
& ((1 - d) (fHR + fLR) + (p (fHR + fLR) + (1 - p) (fHP + fLP)) d (1 - c)) \\
\text{wHP}\phi[\tau_]&:= \\
& ((1 - d) (fHP)) / \\
& ((1 - d) (fHP + fLP) + (p (fHR + fLR) + (1 - p) (fHP + fLP)) d (1 - c)) \\
\text{wLP}\phi[\tau_]&:= \\
& ((1 - d) (fLP)) / \\
& ((1 - d) (fHP + fLP) + (p (fHR + fLR) + (1 - p) (fHP + fLP)) d (1 - c)) \\
\text{vHR}\phi[\tau_]&:= \\
& \text{wHR}\phi[\tau] ((1 - p + p \tau) (vHR[\tau] + vLR[\tau]) + (1 - (1 - p + p \tau)) (vHP[\tau] + vLP[\tau])) \\
\text{vLR}\phi[\tau_]&:= \\
& \text{wLR}\phi[\tau] ((1 - p + p \tau) (vHR[\tau] + vLR[\tau]) + (1 - (1 - p + p \tau)) (vHP[\tau] + vLP[\tau])) \\
\text{vHP}\phi[\tau_]&:= \\
& \text{wHP}\phi[\tau] ((1 - (p + \tau - p \tau)) (vHR[\tau] + vLR[\tau]) + (p + \tau - p \tau) (vHP[\tau] + vLP[\tau]))
\end{aligned}$$

$$\begin{aligned} \mathbf{vLP}\phi[\tau_-] &:= \\ \mathbf{wLP}\phi[\tau] &((1 - (p + \tau - p\tau)) (\mathbf{vHR}[\tau] + \mathbf{vLR}[\tau]) + (p + \tau - p\tau) (\mathbf{vHP}[\tau] + \mathbf{vLP}[\tau])) \end{aligned}$$

$$\begin{aligned} \omega\mathbf{R}[\tau_-] &:= \\ (1 - d) &/ ((1 - d) (\mathbf{fHR} + \mathbf{fLR}) + (p (\mathbf{fHR} + \mathbf{fLR}) + (1 - p) (\mathbf{fHP} + \mathbf{fLP})) d (1 - c)) \end{aligned}$$

$$\begin{aligned} \omega\mathbf{P}[\tau_-] &:= \\ (1 - d) &/ ((1 - d) (\mathbf{fHP} + \mathbf{fLP}) + (p (\mathbf{fHR} + \mathbf{fLR}) + (1 - p) (\mathbf{fHP} + \mathbf{fLP})) d (1 - c)) \end{aligned}$$

$$\begin{aligned} \mathbf{SHR}[\tau_-] &:= -\frac{\mathbf{vHR}[\tau]}{\mathbf{fHR}} \mathbf{cHR} + \left(\mathbf{bHR} \frac{\mathbf{vHR}[\tau]}{\mathbf{fHR}} + \mathbf{bLR} \frac{\mathbf{vLR}[\tau]}{\mathbf{fLR}} \mathbf{rR}[\tau] \right) - \\ &(\mathbf{bHR} + \mathbf{bLR} - \mathbf{cHR}) \omega\mathbf{R}[\tau] (\mathbf{vHR}\phi[\tau] + \mathbf{vLR}\phi[\tau] \mathbf{rR}[\tau]); \\ \mathbf{SLR}[\tau_-] &:= -\frac{\mathbf{vLR}[\tau]}{\mathbf{fLR}} \mathbf{cLR} + \left(\mathbf{bHR} \frac{\mathbf{vHR}[\tau]}{\mathbf{fHR}} \mathbf{rR}[\tau] + \mathbf{bLR} \frac{\mathbf{vLR}[\tau]}{\mathbf{fLR}} \right) - \\ &(\mathbf{bHR} + \mathbf{bLR} - \mathbf{cLR}) \omega\mathbf{R}[\tau] (\mathbf{vHR}\phi[\tau] \mathbf{rR}[\tau] + \mathbf{vLR}\phi[\tau]); \end{aligned}$$

$$\begin{aligned} \mathbf{SHP}[\tau_-] &:= -\frac{\mathbf{vHP}[\tau]}{\mathbf{fHP}} \mathbf{cHP} + \left(\mathbf{bHP} \frac{\mathbf{vHP}[\tau]}{\mathbf{fHP}} + \mathbf{bLP} \frac{\mathbf{vLP}[\tau]}{\mathbf{fLP}} \mathbf{rP}[\tau] \right) - \\ &(\mathbf{bHP} + \mathbf{bLP} - \mathbf{cHP}) \omega\mathbf{P}[\tau] (\mathbf{vHP}\phi[\tau] + \mathbf{vLP}\phi[\tau] \mathbf{rP}[\tau]); \\ \mathbf{SLP}[\tau_-] &:= -\frac{\mathbf{vLP}[\tau]}{\mathbf{fLP}} \mathbf{cLP} + \left(\mathbf{bHP} \frac{\mathbf{vHP}[\tau]}{\mathbf{fHP}} \mathbf{rP}[\tau] + \mathbf{bLP} \frac{\mathbf{vLP}[\tau]}{\mathbf{fLP}} \right) - \\ &(\mathbf{bHP} + \mathbf{bLP} - \mathbf{cLP}) \omega\mathbf{P}[\tau] (\mathbf{vHP}\phi[\tau] \mathbf{rP}[\tau] + \mathbf{vLP}\phi[\tau]); \end{aligned}$$

In[9497]:=

```

Clear[zHR, zLR, zHP, zLP]
listHR = {};
listLR = {};
listHP = {};
listLP = {};
zHR = 0.1; zLR = 0.1; zHP = 0.1; zLP = 0.1;
Do[
  aux = N[FindRoot[{SHR[ $\tau$ ] == 0, SLR[ $\tau$ ] == 0, SHP[ $\tau$ ] == 0, SLP[ $\tau$ ] == 0},
    {{xHR, zHR}, {xLR, zLR}, {xHP, zHP}, {xLP, zLP}}]]];
  listHR = Append[listHR, xHR /. aux];
  listLR = Append[listLR, xLR /. aux];
  listHP = Append[listHP, xHP /. aux];
  listLP = Append[listLP, xLP /. aux];
  zHR = xHR /. aux;
  zLR = xLR /. aux;
  zHP = xHP /. aux;
  zLP = xLP /. aux
, { $\tau$ , - $\frac{99}{100}$ ,  $\frac{99}{100}$ ,  $\frac{1}{50}$ }]

harmingHR = ListInterpolation[listHR, {{- $\frac{99}{100}$ ,  $\frac{99}{100}$ }}]
harmingLR = ListInterpolation[listLR, {{- $\frac{99}{100}$ ,  $\frac{99}{100}$ }}]
harmingHP = ListInterpolation[listHP, {{- $\frac{99}{100}$ ,  $\frac{99}{100}$ }}]
harmingLP = ListInterpolation[listLP, {{- $\frac{99}{100}$ ,  $\frac{99}{100}$ }}]

```

Out[9504]= InterpolatingFunction[

Domain: {{-0.99, 0.99}}
Output: scalar

]

Out[9505]= InterpolatingFunction[

Domain: {{-0.99, 0.99}}
Output: scalar

]

Out[9506]= InterpolatingFunction[

Domain: {{-0.99, 0.99}}
Output: scalar

]

Out[9507]= InterpolatingFunction[

Domain: {{-0.99, 0.99}}
Output: scalar

]

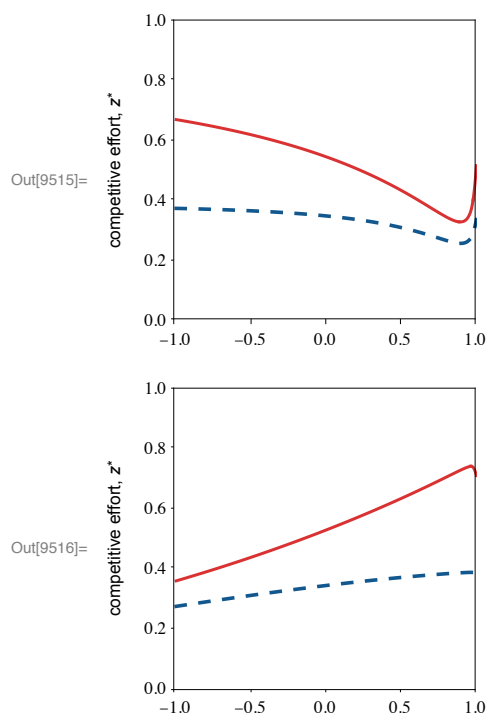
In[9508]:=

```

colour = {"#DA3232", "#12588E"}, {"#DA3232", "#12588E"};
style1 = {Directive[RGBColor[colour[[1]][[1]]], Thickness[0.01]],
  Directive[RGBColor[colour[[1]][[2]]], Dashing[{0.03, 0.05}],
  Thickness[0.012]}};
hlabel = {{-1, "-1.0", {0, 0.01}}, {-0.5, "-0.5", {0, 0.01}},
  {0, "0.0", {0, 0.01}}, {0.5, "0.5", {0, 0.01}},
  {1.0, "1.0", {0, 0.01}}};
vlabel = Join[{{0.0, "0.0", {0, 0.01}}},
  Table[{i, ToString[N[i]], {0, 0.01}}, {i,  $\frac{2}{10}$ , .8,  $\frac{2}{10}$ }],
  {{1, "1.0", {0, 0.01}}}]];
pticks = Join[Table[{i, ToString[i], {0, 0.01}}, {i, -0.8, -0.2, 0.2}],
  {{0, "0.0", {0, 0.01}}},
  Table[{i, ToString[i], {0, 0.01}}, {i, 0.2, 0.8, 0.2}]];
vlabelR = Join[{{0.0, "0.0", {0, 0.01}}},
  Table[{i, ToString[N[i]], {0, 0.01}}, {i,  $\frac{2}{10}$ , .8,  $\frac{2}{10}$ }],
  {{1, "1.0", {0, 0.01}}},
  Table[{i, ToString[N[i]], {0, 0.01}}, {i,  $\frac{12}{10}$ ,  $\frac{20}{10}$ ,  $\frac{2}{10}$ }}]];
vlabelP = Join[{{0.0, "0.00", {0, 0.01}}},
  Table[{i, ToString[N[i]], {0, 0.01}}, {i,  $\frac{2}{100}$ , .08,  $\frac{2}{100}$ }],
  {{0.1, "0.10", {0, 0.01}}},
  Table[{i, ToString[N[i]], {0, 0.01}}, {i,  $\frac{12}{100}$ ,  $\frac{20}{100}$ ,  $\frac{2}{100}$ }}]];

Plot[{harmingHR[ $\tau$ ], harmingLR[ $\tau$ ]}, { $\tau$ , -1, 1},
  PlotRange → {{-1, 1}, {0, 1}}, Frame → True, AxesOrigin → {-1, 0},
  FrameTicks → {{vlabel, None}, {hlabel, None}},
  FrameLabel → {{Text[Style["competitive effort, z*"]], None},
  {None, None}}, PlotStyle → style1, AspectRatio → 1, ImageSize → 200]
Plot[{harmingHP[ $\tau$ ], harmingLP[ $\tau$ ]}, { $\tau$ , -1, 1},
  PlotRange → {{-1, 1}, {0, 1}}, Frame → True, AxesOrigin → {-1, 0},
  FrameTicks → {{vlabel, None}, {hlabel, None}},
  FrameLabel → {{Text[Style["competitive effort, z*"]], None},
  {None, None}}, PlotStyle → style1, AspectRatio → 1, ImageSize → 200]

```



Convergence Stability (CS)

```

c = 0.9;
d = 0.1;
 $\mu_R = 1$ ;
 $\mu_P = 0.1$ ;
qHR = 1;
qLR = 0.1;
qHP = 1;
qLP = 0.1;

```

$$f_{HR} := \mu_R \left(q_{HR} + \frac{x_{HR}}{\frac{1}{2} (x_{HR} + x_{LR})} \left(1 - \frac{1}{2} (x_{HR} + x_{LR}) \right) \right);$$

$$f_{LR} := \mu_R \left(q_{LR} + \frac{x_{LR}}{\frac{1}{2} (x_{HR} + x_{LR})} \left(1 - \frac{1}{2} (x_{HR} + x_{LR}) \right) \right);$$

$$f_{HP} := \mu_P \left(q_{HP} + \frac{x_{HP}}{\frac{1}{2} (x_{HP} + x_{LP})} \left(1 - \frac{1}{2} (x_{HP} + x_{LP}) \right) \right);$$

$$f_{LP} := \mu_P \left(q_{LP} + \frac{x_{LP}}{\frac{1}{2} (x_{HP} + x_{LP})} \left(1 - \frac{1}{2} (x_{HP} + x_{LP}) \right) \right);$$

$$sR = \frac{fLR}{fHR}; sP = \frac{fLP}{fHP}; \sigma = \frac{fHP}{fHR};$$

$$cHR := -\frac{1}{\frac{1}{2}} \left(1 - \frac{1}{2} (xHR + xLR) \right);$$

$$bHR := -\frac{xHR}{\frac{1}{2} (xHR + xLR)};$$

$$cLR := -\frac{1}{\frac{1}{2}} \left(1 - \frac{1}{2} (xHR + xLR) \right);$$

$$bLR := -\frac{xLR}{\frac{1}{2} (xHR + xLR)};$$

$$cHP := -\frac{1}{\frac{1}{2}} \left(1 - \frac{1}{2} (xHP + xLP) \right);$$

$$bHP := -\frac{xHP}{\frac{1}{2} (xHP + xLP)};$$

$$cLP := -\frac{1}{\frac{1}{2}} \left(1 - \frac{1}{2} (xHP + xLP) \right);$$

$$bLP := -\frac{xLP}{\frac{1}{2} (xHP + xLP)};$$

$$vHR[\tau_] := 1$$

$$vLR[\tau_] :=$$

$$\begin{aligned} & \left(sR \right. \\ & \quad \left(-2 (1 + sP) \sigma (-1 + \tau) - \right. \\ & \quad \quad 2 d (-1 - sR + \sigma + sP \sigma + c (1 + sR - (1 + sP) \sigma (-1 + \tau)) - \sigma \tau - sP \sigma \tau) + \\ & \quad \quad \left. d^2 (-1 - sR + \sigma + sP \sigma + c^2 (1 + sR + \sigma + sP \sigma) - 2 c (1 + sP) \sigma \tau) \right) \Big) / \\ & \left(sP \sigma \right. \\ & \quad \left(-2 (1 + sR) (-1 + \tau) + \right. \\ & \quad \quad d^2 (1 + sR - \sigma - sP \sigma + c^2 (1 + sR + \sigma + sP \sigma) - 2 c (1 + sR) \tau) + \\ & \quad \quad \left. 2 d (-1 + \sigma + sP \sigma + sR (-1 + \tau) + \tau - c (1 + sR + \sigma + sP \sigma - \tau - sR \tau)) \right) \Big) / \\ & \left(-2 (1 + sP) \sigma (-1 + \tau) - \right. \\ & \quad \left. 2 d (-1 - sR + \sigma + sP \sigma + c (1 + sR - (1 + sP) \sigma (-1 + \tau)) - \sigma \tau - sP \sigma \tau) + \right. \\ & \quad \left. d^2 (-1 - sR + \sigma + sP \sigma + c^2 (1 + sR + \sigma + sP \sigma) - 2 c (1 + sP) \sigma \tau) \right) \Big) / \end{aligned}$$

```

    (sP σ
      (-2 (1 + sR) (-1 + τ) +
        d2 (1 + sR - σ - sP σ + c2 (1 + sR + σ + sP σ) - 2 c (1 + sR) τ) +
        2 d (-1 + σ + sP σ + sR (-1 + τ) + τ - c (1 + sR + σ + sP σ - τ - sR τ)))))
vHP[τ_] :=
  1/
  sP /
  (-2 (1 + sP) σ (-1 + τ) -
    2 d (-1 - sR + σ + sP σ + c (1 + sR - (1 + sP) σ (-1 + τ)) - σ τ - sP σ τ) +
    d2 (-1 - sR + σ + sP σ + c2 (1 + sR + σ + sP σ) - 2 c (1 + sP) σ τ)) /
  (sP σ
    (-2 (1 + sR) (-1 + τ) +
      d2 (1 + sR - σ - sP σ + c2 (1 + sR + σ + sP σ) - 2 c (1 + sR) τ) +
      2 d (-1 + σ + sP σ + sR (-1 + τ) + τ - c (1 + sR + σ + sP σ - τ - sR τ)))))
vLP[τ_] :=
  1 /
  (-2 (1 + sP) σ (-1 + τ) -
    2 d (-1 - sR + σ + sP σ + c (1 + sR - (1 + sP) σ (-1 + τ)) - σ τ - sP σ τ) +
    d2 (-1 - sR + σ + sP σ + c2 (1 + sR + σ + sP σ) - 2 c (1 + sP) σ τ)) /
  (sP σ
    (-2 (1 + sR) (-1 + τ) +
      d2 (1 + sR - σ - sP σ + c2 (1 + sR + σ + sP σ) - 2 c (1 + sR) τ) +
      2 d (-1 + σ + sP σ + sR (-1 + τ) + τ - c (1 + sR + σ + sP σ - τ - sR τ)))))

wHRφ[τ_] :=
  ((1 - d) (fHR)) /
  ((1 - d) (fHR + fLR) + (p (fHR + fLR) + (1 - p) (fHP + fLP)) d (1 - c))
wLRφ[τ_] :=
  ((1 - d) (fLR)) /
  ((1 - d) (fHR + fLR) + (p (fHR + fLR) + (1 - p) (fHP + fLP)) d (1 - c))
wHPφ[τ_] :=
  ((1 - d) (fHP)) /
  ((1 - d) (fHP + fLP) + (p (fHR + fLR) + (1 - p) (fHP + fLP)) d (1 - c))
wLPφ[τ_] :=
  ((1 - d) (fLP)) /
  ((1 - d) (fHP + fLP) + (p (fHR + fLR) + (1 - p) (fHP + fLP)) d (1 - c))

vHRφ[τ_] :=
  wHRφ[τ] ((1 - p + p τ) (vHR[τ] + vLR[τ]) + (1 - (1 - p + p τ)) (vHP[τ] + vLP[τ]))
vLRφ[τ_] :=
  wLRφ[τ] ((1 - p + p τ) (vHR[τ] + vLR[τ]) + (1 - (1 - p + p τ)) (vHP[τ] + vLP[τ]))

```

```

vHP[τ_] :=
wHP[τ] ((1 - (p + τ - p τ)) (vHR[τ] + vLR[τ]) + (p + τ - p τ) (vHP[τ] + vLP[τ]))
vLP[τ_] :=
wLP[τ] ((1 - (p + τ - p τ)) (vHR[τ] + vLR[τ]) + (p + τ - p τ) (vHP[τ] + vLP[τ]))

```

```

ωR[τ_] :=
(1 - d) / ((1 - d) (fHR + fLR) + (p (fHR + fLR) + (1 - p) (fHP + fLP)) d (1 - c))
ωP[τ_] :=
(1 - d) / ((1 - d) (fHP + fLP) + (p (fHR + fLR) + (1 - p) (fHP + fLP)) d (1 - c))

```

```

SHR[τ_] := - (vHR[τ] / fHR) cHR + (bHR (vHR[τ] / fHR) + bLR (vLR[τ] / fLR) rR[τ]) -
(bHR + bLR - cHR) ωR[τ] (vHR[τ] + vLR[τ] rR[τ]);
SLR[τ_] := - (vLR[τ] / fLR) cLR + (bHR (vHR[τ] / fHR) rR[τ] + bLR (vLR[τ] / fLR)) -
(bHR + bLR - cLR) ωR[τ] (vHR[τ] rR[τ] + vLR[τ]);

```

```

SHP[τ_] := - (vHP[τ] / fHP) cHP + (bHP (vHP[τ] / fHP) + bLP (vLP[τ] / fLP) rP[τ]) -
(bHP + bLP - cHP) ωP[τ] (vHP[τ] + vLP[τ] rP[τ]);
SLP[τ_] := - (vLP[τ] / fLP) cLP + (bHP (vHP[τ] / fHP) rP[τ] + bLP (vLP[τ] / fLP)) -
(bHP + bLP - cLP) ωP[τ] (vHP[τ] rP[τ] + vLP[τ]);

```

```

eig =
Table[
Re[
Eigenvalues[

$$\begin{pmatrix} D[SHR[\tau], xHR] & D[SHR[\tau], xLR] & D[SHR[\tau], xHP] & D[SHR[\tau], xLP] \\ D[SLR[\tau], xHR] & D[SLR[\tau], xLR] & D[SLR[\tau], xHP] & D[SLR[\tau], xLP] \\ D[SHP[\tau], xHR] & D[SHP[\tau], xLR] & D[SHP[\tau], xHP] & D[SHP[\tau], xLP] \\ D[SLP[\tau], xHR] & D[SLP[\tau], xLR] & D[SLP[\tau], xHP] & D[SLP[\tau], xLP] \end{pmatrix} \cdot$$

{xHR → harmingHR[τ], xLR → harmingLR[τ], xHP → harmingHP[τ],
xLP → harmingLP[τ]}]], {τ, - 99/100, 99/100, 1/50}]]

```

```

Out[9549]= {{-0.732379, -0.732379, -0.121681, -0.121681},
{-0.736732, -0.736732, -0.121595, -0.121595},
{-0.74114, -0.74114, -0.121506, -0.121506},
{-0.745604, -0.745604, -0.121415, -0.121415},
{-0.750125, -0.750125, -0.121321, -0.121321},
{-0.754705, -0.754705, -0.121225, -0.121225},

```

```

{-0.759343, -0.759343, -0.121125, -0.121125},
{-0.764041, -0.764041, -0.121023, -0.121023},
{-0.7688, -0.7688, -0.120917, -0.120917},
{-0.773619, -0.773619, -0.120809, -0.120809},
{-0.778501, -0.778501, -0.120697, -0.120697},
{-0.783446, -0.783446, -0.120582, -0.120582},
{-0.788454, -0.788454, -0.120464, -0.120464},
{-0.793527, -0.793527, -0.120343, -0.120343},
{-0.798666, -0.798666, -0.120218, -0.120218},
{-0.803871, -0.803871, -0.12009, -0.12009},
{-0.809143, -0.809143, -0.119958, -0.119958},
{-0.814483, -0.814483, -0.119822, -0.119822},
{-0.819891, -0.819891, -0.119683, -0.119683},
{-0.825369, -0.825369, -0.119539, -0.119539},
{-0.830918, -0.830918, -0.119392, -0.119392},
{-0.836538, -0.836538, -0.11924, -0.11924},
{-0.842229, -0.842229, -0.119084, -0.119084},
{-0.847994, -0.847994, -0.118924, -0.118924},
{-0.853831, -0.853831, -0.11876, -0.11876},
{-0.859743, -0.859743, -0.118591, -0.118591},
{-0.86573, -0.86573, -0.118417, -0.118417},
{-0.871792, -0.871792, -0.118238, -0.118238},
{-0.87793, -0.87793, -0.118054, -0.118054},
{-0.884145, -0.884145, -0.117865, -0.117865},
{-0.890437, -0.890437, -0.117671, -0.117671},
{-0.896806, -0.896806, -0.117471, -0.117471},
{-0.903254, -0.903254, -0.117266, -0.117266},
{-0.90978, -0.90978, -0.117055, -0.117055},
{-0.916384, -0.916384, -0.116838, -0.116838},
{-0.923068, -0.923068, -0.116615, -0.116615},
{-0.92983, -0.92983, -0.116385, -0.116385},
{-0.936671, -0.936671, -0.116149, -0.116149},
{-0.943591, -0.943591, -0.115906, -0.115906},
{-0.950589, -0.950589, -0.115657, -0.115657},
{-0.957666, -0.957666, -0.115399, -0.115399},
{-0.96482, -0.96482, -0.115135, -0.115135},
{-0.97205, -0.97205, -0.114862, -0.114862},
{-0.979356, -0.979356, -0.114582, -0.114582},
{-0.986737, -0.986737, -0.114293, -0.114293},
{-0.994191, -0.994191, -0.113996, -0.113996},
{-1.00172, -1.00172, -0.113689, -0.113689},
{-1.00931, -1.00931, -0.113373, -0.113373},
{-1.01697, -1.01697, -0.113048, -0.113048},
{-1.0247, -1.0247, -0.112712, -0.112712},
{-1.03248, -1.03248, -0.112366, -0.112366},
{-1.04033, -1.04033, -0.112009, -0.112009},
{-1.04822, -1.04822, -0.111641, -0.111641},
{-1.05617, -1.05617, -0.11126, -0.11126},
{-1.06416, -1.06416, -0.110867, -0.110867},
{-1.07218, -1.07218, -0.110461, -0.110461},
{-1.08023, -1.08023, -0.110041, -0.110041},

```

```

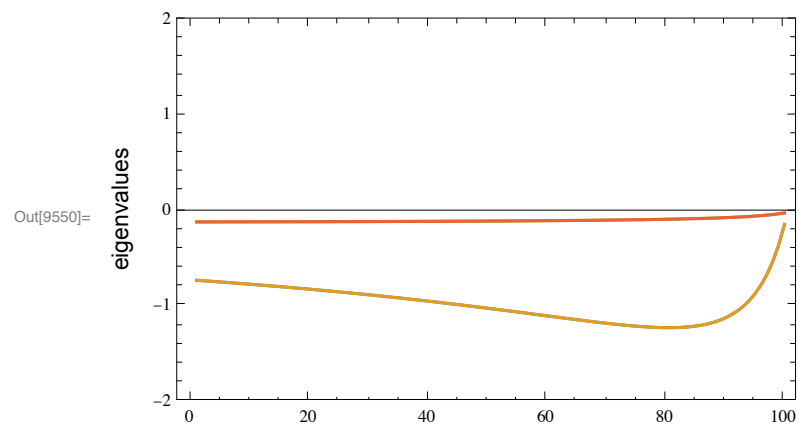
{-1.0883, -1.0883, -0.109607, -0.109607},
{-1.09639, -1.09639, -0.109158, -0.109158},
{-1.10447, -1.10447, -0.108693, -0.108693},
{-1.11255, -1.11255, -0.108211, -0.108211},
{-1.1206, -1.1206, -0.107712, -0.107712},
{-1.12861, -1.12861, -0.107193, -0.107193},
{-1.13656, -1.13656, -0.106656, -0.106656},
{-1.14444, -1.14444, -0.106097, -0.106097},
{-1.15222, -1.15222, -0.105517, -0.105517},
{-1.15988, -1.15988, -0.104913, -0.104913},
{-1.16739, -1.16739, -0.104284, -0.104284},
{-1.17472, -1.17472, -0.103628, -0.103628},
{-1.18182, -1.18182, -0.102944, -0.102944},
{-1.18867, -1.18867, -0.102229, -0.102229},
{-1.19521, -1.19521, -0.101481, -0.101481},
{-1.20139, -1.20139, -0.100699, -0.100699},
{-1.20715, -1.20715, -0.0998779, -0.0998779},
{-1.21241, -1.21241, -0.099016, -0.099016},
{-1.2171, -1.2171, -0.0981096, -0.0981096},
{-1.22113, -1.22113, -0.0971547, -0.0971547},
{-1.22438, -1.22438, -0.096147, -0.096147},
{-1.22673, -1.22673, -0.0950815, -0.0950815},
{-1.22805, -1.22805, -0.0939525, -0.0939525},
{-1.22815, -1.22815, -0.0927536, -0.0927536},
{-1.22685, -1.22685, -0.0914773, -0.0914773},
{-1.22392, -1.22392, -0.0901152, -0.0901152},
{-1.21908, -1.21908, -0.0886573, -0.0886573},
{-1.21201, -1.21201, -0.0870921, -0.0870921},
{-1.20234, -1.20234, -0.0854061, -0.0854061},
{-1.1896, -1.1896, -0.0835834, -0.0835834},
{-1.17324, -1.17324, -0.081605, -0.081605},
{-1.15261, -1.15261, -0.0794481, -0.0794481},
{-1.12687, -1.12687, -0.0770855, -0.0770855},
{-1.09504, -1.09504, -0.0744838, -0.0744838},
{-1.05586, -1.05586, -0.0716018, -0.0716018},
{-1.00777, -1.00777, -0.0683887, -0.0683887},
{-0.948793, -0.948793, -0.06478, -0.06478},
{-0.876372, -0.876372, -0.0606936, -0.0606936},
{-0.78719, -0.78719, -0.0560229, -0.0560229},
{-0.676856, -0.676856, -0.050627, -0.050627},
{-0.539465, -0.539465, -0.0443156, -0.0443156},
{-0.366977, -0.366977, -0.0368241, -0.0368241},
{-0.148578, -0.148578, -0.0277734, -0.0277734}}

```

```

In[9550]:= ListLinePlot[Transpose[eig], PlotRange → {-2, 2}, Frame → True,
FrameLabel →
{{Text[Style["eigenvalues", FontSize → 12, FontFamily → "Arial"]],
None}, {None, None}}]

```



patch size, $n = 4$

code

```

In[1]:= Clear[w, vn, v, pn, p, rn, r, A, fecundity, z, cost, benefit,
  social, dyna, Smatrix, P, f, wφ, wδ]
n = 4; (* patch size *)
a = 2; (* number of high-quality breeders *)
t = 2; (* two qualities of individuals *)
NP = 2; (* patch types - resource-rich and resource-poor *)
tmax = 800;
c =  $\frac{5}{10}$ ; (* cost of dispersal *)
d =  $\frac{1}{10}$ ; (* dispersal rate *)
GI[i_, k_] := If[i == 1, 1,  $\frac{1}{10}$ ] (* baseline fecundity *)
GP[1] := 1; (* resource-availability in resource-rich patches *)
GP[2] :=  $\frac{1}{10}$ ; (* resource-availability in resource-poor patches *)

cost[i_, q_] := -  $\frac{(1 - \frac{1}{n} \sum_{j=1}^t a z[[q]][[j]])}{\frac{1}{n}}$  (* cost of the behaviour *)

benefit[i_, q_] := -  $\frac{n z[[q]][[i]]}{\sum_{j=1}^t a z[[q]][[j]]}$  (* benefit of the behaviour *)

z = Table[ $\frac{1}{10}$ , {q, 1, NP}, {i, 1, t}]
(* initial competitive effort values *)
dyna = {z};

f[i_, k_] :=
  GP[k] *  $\left( GI[i, k] + \frac{z[[k]][[i]]}{\frac{1}{n} \sum_{j=1}^t (a z[[k]][[j]])} \left( 1 - \frac{1}{n} \sum_{j=1}^t (a z[[k]][[j]]) \right) \right)$ 
(* fecundity of each individual *)

upperH = {}; (* auxiliary vectors *)
lowerH = {};
upperL = {};
lowerL = {};

upperVH = {};
lowerVH = {};

```

```
upperVL = {};
```

```
lowerVL = {};
```

```
upperZH = {};
```

```
lowerZH = {};
```

```
upperZL = {};
```

```
lowerZL = {};
```

```
rH = {};
```

```
rL = {};
```

```
 $\theta = \frac{1}{2};$  (* proportion of resource-rich patches *)
```

```
 $\tau = -\frac{99}{100};$  (* initial value of temporal correlation *)
```

```
Do[
```

$$P = \begin{pmatrix} \theta + \tau - \theta \tau & 1 - (\theta + \tau - \theta \tau) \\ 1 - (1 - \theta + \theta \tau) & 1 - \theta + \theta \tau \end{pmatrix};$$

```
Do[
```

$$pn[i_]:= \sum_{j=1}^{NP} p[j] P[[i]][[j]];$$

```
solp = NSolve[Join[Table[pn[i] == p[i], {i, 1, NP}], { $\left(\sum_{i=1}^{NP} p[i]\right) == 1$ }],
```

```
Table[p[i], {i, 1, NP}]]][[1]];
```

```
Table[{p[i] == p[i] /. solp}, {i, 1, NP}];
```

$$w\phi[i_, j_] := ((1 - d) f[i, j]) / \left(\left(\sum_{l=1}^t a f[l, j] \right) (1 - d) + \left(\sum_{k=1}^{NP} p[k] \sum_{l=1}^t a f[l, k] \right) d (1 - c) \right);$$

```
wδ[i_, j_, e_] :=
```

$$(f[i, j] d (1 - c)) /$$

$$\left(\left(\sum_{l=1}^t a f[l, e] \right) (1-d) + \left(\sum_{k=1}^{NP} p[k] \sum_{l=1}^t a f[l, k] \right) d (1-c) \right);$$

$w[i_, j_, u_, q_] :=$

$$w\phi[i, j] P[[j]][[q]] + \sum_{e=1}^{NP} p[e] w\delta[i, j, e] P[[e]][[q]];$$

$vn[i_, j_] := \text{If}[i == j == 1, 1, \sum_{q=1}^{NP} \sum_{u=1}^t (w[i, j, u, q] a v[u, q])];$

$\text{solrv} = \text{NSolve}[\text{Flatten}[\text{Table}[vn[i, q] == v[i, q], \{i, 1, t\}, \{q, 1, NP\}]],$
 $\text{Flatten}[\text{Table}[v[i, q], \{i, 1, t\}, \{q, 1, NP\}]]][[1]]];$

$\text{Table}[\{v[i, q] = v[i, q] /. \text{solrv}\}, \{i, 1, t\}, \{q, 1, NP\}];$

$\omega[i_, q_] :=$

$$(1-d) / \left(\left(\sum_{l=1}^t a f[l, q] \right) (1-d) + \left(\sum_{k=1}^{NP} p[k] \sum_{l=1}^t a f[l, k] \right) d (1-c) \right);$$

$vp[y_, e_] := \sum_{q=1}^{NP} \sum_{u=1}^t w\phi[y, e] a P[[e]][[q]] v[u, q];$

$\varphi[j_] :=$

$$\left(\left(\left(\sum_{i=1}^t (a f[i, j]) \right) (1-d) \right) / \right. \\ \left. \left(\left(\sum_{i=1}^t (a f[i, j]) \right) (1-d) + \left(\sum_{k=1}^{NP} p[k] \sum_{i=1}^t (a f[i, k]) \right) d (1-c) \right) \right)^2;$$

$rn[q_] :=$

$$\sum_{k=1}^{NP} P[[k]][[q]] \varphi[k] \\ \left(\left(a \sum_{l=1}^t \left(\frac{f[l, k]}{\sum_{e=1}^t a f[e, k]} \right)^2 \right) + \left(1 - \left(a \sum_{l=1}^t \left(\frac{f[l, k]}{\sum_{e=1}^t a f[e, k]} \right)^2 \right) \right) r[k] \right);$$

$\text{solg} = \text{NSolve}[\text{Flatten}[\text{Table}[rn[q] == r[q], \{q, 1, NP\}]],$

$\text{Flatten}[\text{Table}[r[q], \{q, 1, NP\}]]];$

$\text{Table}[\{r[q] = r[q] /. \text{solg}][[1]]\}, \{q, 1, NP\}];$

$$\rho[i_, k_, q_] := \text{If}[i == k, \frac{1}{a} + \frac{a-1}{a} r[q], r[q]];$$

$$S[i_, q_] := -\frac{v[i, q]}{f[i, q]} \text{cost}[i, q] + \sum_{k=1}^t a \text{benefit}[k, q] \frac{v[k, q]}{f[k, q]} \rho[i, k, q] -$$

$$\left(\sum_{k=1}^t \text{benefit}[k, q] a \omega[k, q] \right) \left(\sum_{l=1}^t a v[l, q] \rho[i, l, q] \right) +$$

$$\text{cost}[i, q] \omega[i, q] \left(\sum_{k=1}^t a v[k, q] \rho[i, k, q] \right);$$

z =

Clip[

Table[N[Rationalize[z[[q]][[i]] + 0.0015 * Sign[S[i, q]], 0.00001],
{q, 1, NP}, {i, 1, t}], {0.0, 1.0}];

dyna = Append[dyna, z];

Clear[p, pn, r, rn, v, vn, ρ];

If[asd > 19, Break[]], {σ, tmax}];

upperZH = Append[upperZH, {τ, z[[1]][[1]]}];

lowerZH = Append[lowerZH, {τ, z[[1]][[2]]}];

upperZL = Append[upperZL, {τ, z[[2]][[1]]}];

lowerZL = Append[lowerZL, {τ, z[[2]][[2]]}];

Print[τ],

$$\left\{ \tau, -\frac{99}{100} + \frac{10}{100}, \frac{99}{100}, \frac{10}{100} \right\}$$

Out[11]= $\left\{ \left\{ \frac{1}{10}, \frac{1}{10} \right\}, \left\{ \frac{1}{10}, \frac{1}{10} \right\} \right\}$

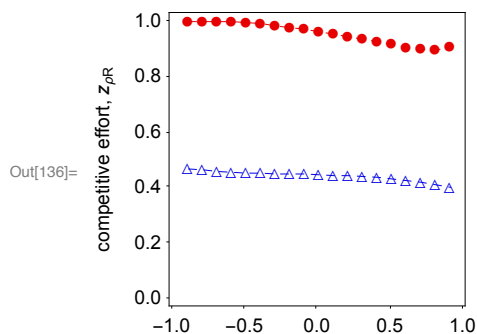
plot - optimal competitive effort strategies

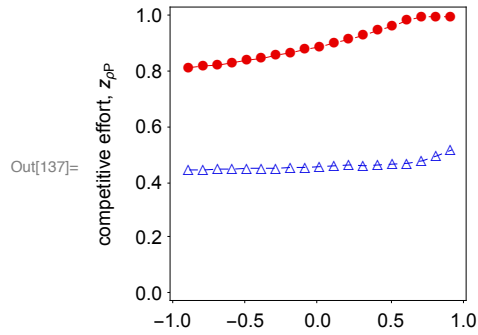
```

In[136]:= panel1 = ListLinePlot[{upperZH, lowerZH},
  PlotRange → {{-1.01, 1.01}, {-0.03, 1.03}}, Frame → True,
  AxesOrigin → {-1, 0}, BaseStyle → {FontSize → 10, FontFamily → "Arial"},
  FrameLabel →
    {{Text[Style["competitive effort,  $z_{pR}$ ", FontSize → 10,
      FontFamily → "Arial"]], None}, {None, None}},
  FrameTicks →
    {{Join[{{0, "0.0"}, {0, 0.01}}],
      Table[{i, ToString[N[i]], {0, 0.01}}, {i, 0.2, 0.8, 0.2}],
      {{1, "1.0"}, {0, 0.01}}]}, None},
    {{{-1, "-1.0"}, {0, 0.01}}, {-0.5, "-0.5"}, {0, 0.01}},
    {0, "0.0"}, {0, 0.01}}, {0.5, "0.5"}, {0, 0.01}},
    {1, "1.0"}, {0, 0.01}}]}, None}},
  PlotStyle → {Directive[RGBColor[.9, 0, 0], Dashed, Thickness[0.001]],
    Directive[RGBColor[0, 0, .9], Dashed, Thickness[0.001]]},
  PlotMarkers → {"●", 8}, {"△", 10}}, ImageSize → 200, AspectRatio → 1]

panel2 = ListLinePlot[{upperZL, lowerZL},
  PlotRange → {{-1.01, 1.01}, {-0.03, 1.03}}, Frame → True,
  AxesOrigin → {-1, 0}, BaseStyle → {FontSize → 10, FontFamily → "Arial"},
  FrameLabel →
    {{Text[Style["competitive effort,  $z_{pP}$ ", FontSize → 10,
      FontFamily → "Arial"]], None}, {None, None}},
  FrameTicks →
    {{Join[{{0, "0.0"}, {0, 0.01}}],
      Table[{i, ToString[N[i]], {0, 0.01}}, {i, 0.2, 0.8, 0.2}],
      {{1, "1.0"}, {0, 0.01}}]}, None},
    {{{-1, "-1.0"}, {0, 0.01}}, {-0.5, "-0.5"}, {0, 0.01}},
    {0, "0.0"}, {0, 0.01}}, {0.5, "0.5"}, {0, 0.01}},
    {1, "1.0"}, {0, 0.01}}]}, None}},
  PlotStyle → {Directive[RGBColor[.9, 0, 0], Dashed, Thickness[0.001]],
    Directive[RGBColor[0, 0, .9], Dashed, Thickness[0.001]]},
  PlotMarkers → {"●", 8}, {"△", 10}}, ImageSize → 200, AspectRatio → 1]

```





upperZH[All, 2]

lowerZH[All, 2]

```
{0.684874, 0.684874, 0.684874, 0.681905, 0.681922, 0.678922,
 0.678922, 0.675926, 0.675926, 0.672922, 0.669935, 0.666937,
 0.660944, 0.657971, 0.651961, 0.642981, 0.630952, 0.612943, 0.59194}
```

```
{0.153996, 0.153996, 0.156997, 0.153996, 0.156997, 0.153996, 0.154, 0.154004, 0.154,
 0.150997, 0.154, 0.154, 0.151007, 0.151007, 0.148, 0.148, 0.145, 0.138996, 0.133005}
```

plot - inequality within patches

In[138]=

fHR =

GP[1] *

$$\left(\text{GI}[1, 1] + \frac{\text{upperZH}[\text{All}, 2]}{\frac{1}{n} (\text{a upperZH}[\text{All}, 2] + \text{a lowerZH}[\text{All}, 2])} \right) \left(1 - \frac{1}{n} (\text{a upperZH}[\text{All}, 2] + \text{a lowerZH}[\text{All}, 2]) \right)$$

fLR =

GP[1] *

$$\left(\text{GI}[2, 1] + \frac{\text{lowerZH}[\text{All}, 2]}{\frac{1}{n} (\text{a upperZH}[\text{All}, 2] + \text{a lowerZH}[\text{All}, 2])} \right) \left(1 - \frac{1}{n} (\text{a upperZH}[\text{All}, 2] + \text{a lowerZH}[\text{All}, 2]) \right)$$

fHP =

GP[2] *

$$\left(\text{GI}[1, 2] + \frac{\text{upperZL}[\text{All}, 2]}{\frac{1}{n} (\text{a upperZL}[\text{All}, 2] + \text{a lowerZL}[\text{All}, 2])} \right) \left(1 - \frac{1}{n} (\text{a upperZL}[\text{All}, 2] + \text{a lowerZL}[\text{All}, 2]) \right)$$

fLP =

GP[2] *

$$\left(\text{GI}[2, 2] + \frac{\text{lowerZL}[\text{All}, 2]}{\frac{1}{n} (\text{a upperZL}[\text{All}, 2] + \text{a lowerZL}[\text{All}, 2])} \right. \\ \left. \left(1 - \frac{1}{n} (\text{a upperZL}[\text{All}, 2] + \text{a lowerZL}[\text{All}, 2]) \right) \right)$$

```

panel3 = ListLinePlot[{{Table[0.9, {i, 1, Length[fLR]}], 1 -  $\frac{\text{fLR}}{\text{fHR}}$ , 1 -  $\frac{\text{fLP}}{\text{fHP}}$ }},
  PlotRange -> {{0, Length[fLR] + 1}, {0.7, 1}}, Frame -> True,
  BaseStyle -> {FontSize -> 10, FontFamily -> "Arial"},
  FrameLabel ->
    {{Text[Style["within-group inequality,  $G_{\sigma}$ ", FontSize -> 10,
      FontFamily -> "Arial"]], None}, {None, None}},
  FrameTicks ->
    {{{{0.7, "0.7", {0, 0.01}}, {0.8, "0.8", {0, 0.01}},
      {0.8, "0.8", {0, 0.01}}, {0.9, "0.9", {0, 0.01}},
      {1.0, "1.0", {0, 0.01}}}}, None},
    {{{1, "-1.0", {0, 0.01}}, {5.5, "-0.5", {0, 0.01}},
      {10, "0.0", {0, 0.01}}, {14.5, "0.5", {0, 0.01}},
      {19, "1.0", {0, 0.01}}}}, None}},
  PlotStyle -> {Directive[RGBColor[0, 0, 0], Dotted, Thickness[0.01]],
    Directive[RGBColor[.9, 0, 0], Dashed, Thickness[0.001]],
    Directive[RGBColor[0, 0, .9], Dashed, Thickness[0.001]]},
  PlotMarkers -> {{{"", 8}, {"●", 8}, {"△", 10}}, ImageSize -> 200,
  AspectRatio -> 1]

```

```

Out[138]= {1.36162, 1.36441, 1.37, 1.37281, 1.37784, 1.37954,
  1.38745, 1.39082, 1.39417, 1.40203, 1.40987, 1.41476,
  1.42254, 1.43033, 1.4411, 1.45347, 1.46271, 1.47364, 1.47682}

```

```

Out[139]= {0.269541, 0.269755, 0.270146, 0.270324, 0.27216, 0.273457,
  0.277509, 0.280143, 0.28281, 0.286949, 0.291096, 0.295216,
  0.299432, 0.303667, 0.307919, 0.313568, 0.316285, 0.31737, 0.31121}

```

```

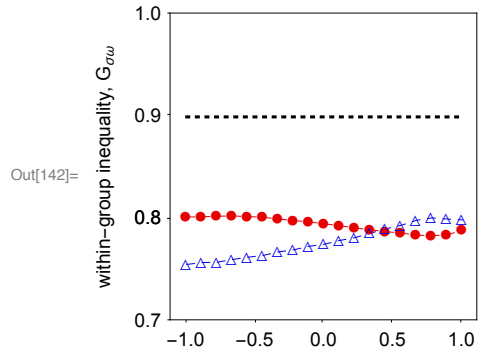
Out[140]= {0.147492, 0.147226, 0.146789, 0.146384, 0.14567, 0.145396,
  0.144831, 0.144101, 0.14352, 0.142783, 0.141736, 0.140835, 0.140341,
  0.139099, 0.138003, 0.136566, 0.135055, 0.133433, 0.131593}

```

```

Out[141]= {0.0360413, 0.0357074, 0.0355461, 0.0350503, 0.0345638, 0.0342425,
  0.0336081, 0.0331366, 0.0325203, 0.0320591, 0.0313027, 0.0307044,
  0.0299962, 0.0291378, 0.02843, 0.0274672, 0.0268572, 0.0266791, 0.0264232}

```



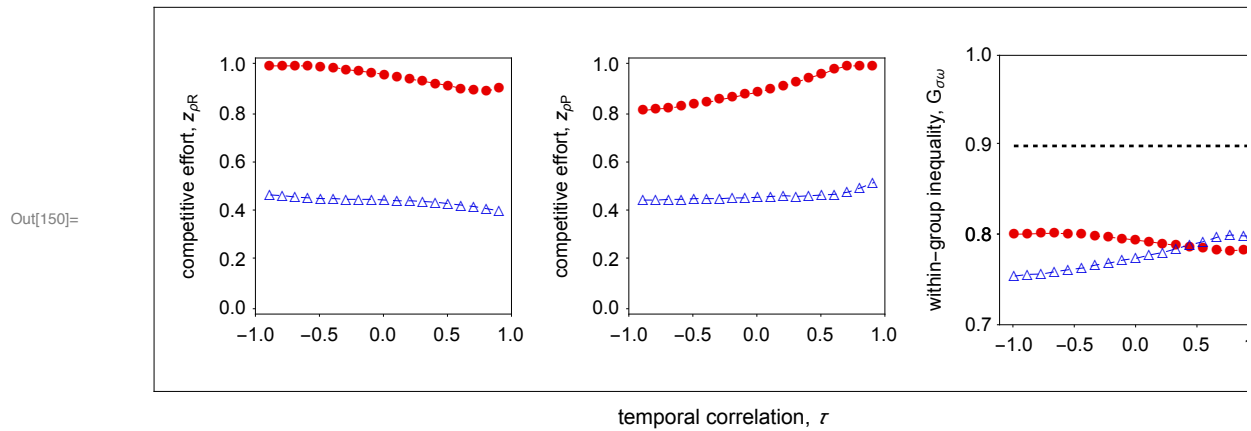
plot - figureI1

```

In[150]:= figureI1 =
  Show[GraphicsGrid[{{panel1, panel2, panel3}}, Frame → True,
    FrameStyle → White], Frame → True, FrameTicks → None,
    FrameLabel →
      {{None, None},
        {Text[Style["temporal correlation,  $\tau$ ", FontSize → 11,
          FontFamily → "Arial"]], None}}]

Export["figureI1.tiff", figureI1, ImageResolution → 250]
Export["figureI1.pdf", figureI1]

```



Out[151]= figureI1.tiff

Out[152]= figureI1.pdf

Early-life acquisition of quality code

```

Clear[q, p, f, d, c, m, i, j, n, w, v, V, r, A, W, w, z, W\phi, cost,
  benefit, SG]

```

$$\tau = \frac{\theta}{10};$$

```

tmax = 300;

d =  $\frac{1}{10}$ ; c =  $\frac{1}{2}$ ; Q =  $\frac{1}{2}$ ; n = 2;

GP[1] := 1; GP[2] :=  $\frac{1}{10}$ ;

GI[1, 1] := 1; GI[2, 1] :=  $\frac{1}{10}$ ;

GI[1, 2] := 1; GI[2, 2] :=  $\frac{1}{10}$ ;

z = Table[ $\frac{6}{10}$ , {k, 1, 2}, {i, 1, 2}, {j, 1, 2}]

o[1, 1] := 2; o[1, 2] := 1; o[2, 1] := 1; o[2, 2] := 2;

q[1, 1] :=  $Q^2$ ; q[1, 2] :=  $2 Q (1 - Q)$ ; q[2, 1] :=  $2 Q (1 - Q)$ ;
q[2, 2] :=  $(1 - Q)^2$ ;

p[1] :=  $\frac{1}{2}$ ; p[2] :=  $\frac{1}{2}$ ;

Prob[1, 1] :=  $\tau$ ;
Prob[1, 2] :=  $1 - \tau$ ;
Prob[2, 1] :=  $1 - \tau$ ;
Prob[2, 2] :=  $\tau$ ;

dyna = {z};

Do[

cost[i_, j_, k_] := -  $\frac{(1 - \frac{1}{n} (z[[k]][[i]][[j]] + z[[k]][[j]][[i]]))}{\frac{1}{n}}$ ;

benefit[i_, j_, k_] := -  $\frac{n z[[k]][[i]][[j]]}{(z[[k]][[i]][[j]] + z[[k]][[j]][[i]])}$ ;

f[i_, j_, k_] :=
GP[k] *
 $\left( GI[i, k] + \frac{z[[k]][[i]][[j]]}{\frac{1}{n} (z[[k]][[i]][[j]] + z[[k]][[j]][[i]])} \right. \\ \left. \left( 1 - \frac{1}{n} (z[[k]][[i]][[j]] + z[[k]][[j]][[i]]) \right) \right)$ ;

```

$$S := \left(\sum_{l=1}^2 p[l] \sum_{e=1}^2 \sum_{u=e}^2 q[e, u] (f[e, u, l] + f[u, e, l]) \right) d (1 - c);$$

$$w\phi[i_, j_, k_] := \frac{f[i, j, k] (1 - d)}{(f[i, j, k] + f[j, i, k]) (1 - d) + S};$$

$$W\phi[i_, j_, k_] := \frac{1 - d}{(f[i, j, k] + f[j, i, k]) (1 - d) + S};$$

$$w\delta[i_, j_, k_, m_, n_, e_] := \frac{f[i, j, k] d (1 - c)}{(f[m, n, e] + f[n, m, e]) (1 - d) + S};$$

$$\begin{aligned} w[i_, j_, a_, e_, u_, b_] := & \left(w\phi[i, j, a] \text{Prob}[a, b] + \right. \\ & \left. \sum_{g=1}^2 p[g] \text{Prob}[g, b] \left(\sum_{l=1}^2 \sum_{m=l}^2 w\delta[i, j, a, l, m, g] q[l, m] \right) \right) o[e, u] \\ & q[e, u]; \end{aligned}$$

```
wmatrix =
Transpose[
  Partition[Flatten[Table[(w[i, j, k, e, u, l]), {k, 1, 2},
    {i, 1, 2}, {j, 1, 2}, {l, 1, 2}, {e, 1, 2}, {u, 1, 2}]], 8]];
```

```
vn[i_, j_, a_] :=
  If[i == j == a == 1, 1, \sum_{b=1}^2 \sum_{e=1}^2 \sum_{u=1}^2 w[i, j, a, e, u, b] v[e, u, b]];
```

```
solv =
NDSolve[Flatten[Table[vn[i, j, a] == v[i, j, a], {a, 1, 2},
  {i, 1, 2}, {j, 1, 2}]],
  Flatten[Table[v[i, j, a], {a, 1, 2}, {i, 1, 2}, {j, 1, 2}]]][[1]];
```

```
Table[{v[i, j, a] = v[i, j, a] /. solv}, {a, 1, 2}, {i, 1, 2},
  {j, 1, 2}];
```

```
Flatten[Table[V[i, j, k] = \frac{v[i, j, k]}{f[i, j, k]}, {k, 1, 2}, {i, 1, 2}, {j, 1, 2}]]];
```

```
rn[i_, j_, k_] :=
```

$$\sum_{g=1}^2 \text{Prob}[g, k] \left(\sum_{l=1}^2 \sum_{m=l}^2 q[l, m] (w\phi[l, m, g]^2 + w\phi[m, l, g]^2 + 2 w\phi[l, m, g] w\phi[m, l, g] r[l, m, g]) \right);$$

```
solg = NSolve[{rn[1, 1, 1] == r[1, 1, 1], rn[1, 2, 1] == r[1, 2, 1],
  rn[2, 2, 1] == r[2, 2, 1], rn[1, 1, 2] == r[1, 1, 2],
  rn[1, 2, 2] == r[1, 2, 2], rn[2, 2, 2] == r[2, 2, 2]},
  {r[1, 1, 1], r[1, 2, 1], r[2, 2, 1], r[1, 1, 2], r[1, 2, 2],
  r[2, 2, 2]}];
```

```
Table[{r[i, j, k] = r[i, j, k] /. solg[[1]]}, {i, 1, 2}, {j, 1, 2},
  {k, 1, 2}];
r[2, 1, 1] = r[1, 2, 1];
r[2, 1, 2] = r[1, 2, 2];
```

```
vφ[i_, j_, a_] :=
```

$$\sum_{g=1}^2 \text{Prob}[a, g] \sum_{l=1}^2 \sum_{m=1}^2 w\phi[i, j, a] o[l, m] q[l, m] v[l, m, g];$$

```
A[i_, j_, k_] :=
```

$$(V[j, i, k] r[i, j, k] - W\phi[i, j, k] (v\phi[i, j, k] + v\phi[j, i, k] r[i, j, k])) / (V[i, j, k] - W\phi[i, j, k] (v\phi[i, j, k] + v\phi[j, i, k] r[i, j, k]));$$

```
SG[i_, j_, k_] := -V[i, j, k] cost[i, j, k] +
  (benefit[i, j, k] V[i, j, k] + benefit[j, i, k] V[j, i, k] r[i, j, k]) -
  ((benefit[i, j, k] + benefit[j, i, k]) W\phi[i, j, k]
  (v\phi[i, j, k] + v\phi[j, i, k] r[i, j, k])) +
  cost[i, j, k] W\phi[i, j, k] (v\phi[i, j, k] + v\phi[j, i, k] r[i, j, k]);
```

```
z =
```

```
Clip[
  Table[N[Rationalize[z[[k]][[i]][[j]] + 0.001 * Sign[SG[i, j, k]],
    0.00001]], {k, 1, 2}, {i, 1, 2}, {j, 1, 2}], {0.001, 0.999}];
```

```
dyna = Append[dyna, z];
```

```
Clear[r, rn, v, vn, rv, v];
```

```

If[t > 19, Break[]], {σ, tmax}]

z111 = Table[dyna[[t]][[1]][[1]][[1]], {t, 1, Length[dyna]};
z121 = Table[dyna[[t]][[1]][[1]][[2]], {t, 1, Length[dyna]};
z211 = Table[dyna[[t]][[1]][[2]][[1]], {t, 1, Length[dyna]};
z221 = Table[dyna[[t]][[1]][[2]][[2]], {t, 1, Length[dyna]};
ListLinePlot[{z111, z121, z211, z221}, PlotRange → {0, 1},
  PlotLegends → Automatic]

z112 = Table[dyna[[t]][[2]][[1]][[1]], {t, 1, Length[dyna]};
z122 = Table[dyna[[t]][[2]][[1]][[2]], {t, 1, Length[dyna]};
z212 = Table[dyna[[t]][[2]][[2]][[1]], {t, 1, Length[dyna]};
z222 = Table[dyna[[t]][[2]][[2]][[2]], {t, 1, Length[dyna]};

ListLinePlot[{z112, z122, z212, z222}, PlotRange → {0, 1},
  PlotLegends → Automatic]

high11 = {Last[z111], Last[z121], Last[z211], Last[z221]}
low11 = {Last[z112], Last[z122], Last[z212], Last[z222]}

plot - optimal levels of competitive effort

high = {high1, high2a, high2, high3a, high3, high4a, high4, high5a,
  high5, high6a, high6, high7a, high7, high8a, high8, high9a,
  high9, high10a, high10, high11a, high11}
low = {low1, low2a, low2, low3a, low3, low4a, low4, low5a, low5,
  low6a, low6, low7a, low7, low8a, low8, low9a, low9, low10a,
  low10, low11a, low11}
base = Table[0.5, {i, 1, Length[high]}]

fig4A =
ListLinePlot[{Reverse[Table[high[[i]][[1]], {i, 1, Length[high]}]],
  Reverse[Table[high[[i]][[2]], {i, 1, Length[high]}]],
  Reverse[Table[high[[i]][[3]], {i, 1, Length[high]}]],
  Reverse[Table[high[[i]][[4]], {i, 1, Length[high]}]]],
PlotRange → {{0.5, Length[high] + 0.5}, {0, 1}}, Frame → True,
FrameLabel →
  {{Text[Style["competitive effort, z*"], FontSize → 11,
    FontFamily → "Arial"]], None}, {None, None}},
FrameTicks →
  {{{{0, "0.0", {0, 0.01}}, {0.2, "0.2", {0, 0.01}},
    {0.4, "0.4", {0, 0.01}}, {0.6, "0.6", {0, 0.01}},
    {0.8, "0.8", {0, 0.01}}, {1, "1.0", {0, 0.01}}}, None},
  {{{1, "-1.0", {0, 0.01}}, {6, "-0.5", {0, 0.01}},

```

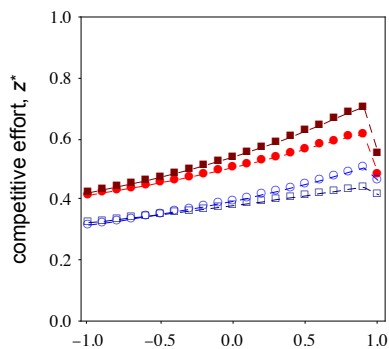
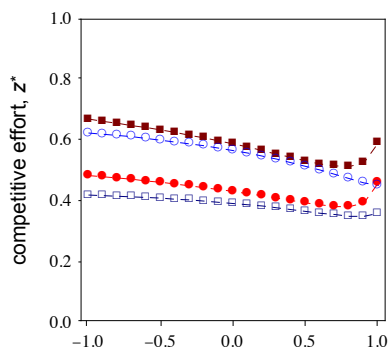
```

{11, "0.0", {0, 0.01}}, {16, "0.5", {0, 0.01}},
{21, "1.0", {0, 0.01}}}, None}},
PlotStyle → {Directive[RGBColor[1, 0, 0], Dashed, Thickness[0.001]],
  Directive[RGBColor[.5, 0, 0], Dashed, Thickness[0.001]],
  Directive[RGBColor[0, 0, .5], Dashed, Thickness[0.001]],
  Directive[RGBColor[0, 0, 1], Dashed, Thickness[0.001]]},
PlotMarkers → {"●", 8}, {"■", 8}, {"□", 8}, {"○", 8}},
AspectRatio → 1, ImageSize → 200]
fig4B =
ListLinePlot[{Reverse[Table[low[[i]][[1]], {i, 1, Length[low]}]],
  Reverse[Table[low[[i]][[2]], {i, 1, Length[low]}]],
  Reverse[Table[low[[i]][[3]], {i, 1, Length[low]}]],
  Reverse[Table[low[[i]][[4]], {i, 1, Length[low]}]]},
PlotRange → {{0.5, Length[high] + 0.5}, {0, 1}}, Frame → True,
FrameLabel →
{{Text[Style["competitive effort, z*", FontSize → 11,
  FontFamily → "Arial"]], None}, {None, None}},
FrameTicks →
{{{0, "0.0", {0, 0.01}}, {0.2, "0.2", {0, 0.01}},
  {0.4, "0.4", {0, 0.01}}, {0.6, "0.6", {0, 0.01}},
  {0.8, "0.8", {0, 0.01}}, {1, "1.0", {0, 0.01}}}, None},
{{1, "-1.0", {0, 0.01}}, {6, "-0.5", {0, 0.01}},
  {11, "0.0", {0, 0.01}}, {16, "0.5", {0, 0.01}},
  {21, "1.0", {0, 0.01}}}, None}},
PlotStyle → {Directive[RGBColor[1, 0, 0], Dashed, Thickness[0.001]],
  Directive[RGBColor[.5, 0, 0], Dashed, Thickness[0.001]],
  Directive[RGBColor[0, 0, .5], Dashed, Thickness[0.001]],
  Directive[RGBColor[0, 0, 1], Dashed, Thickness[0.001]]},
PlotMarkers → {"●", 8}, {"■", 8}, {"□", 8}, {"○", 8}},
AspectRatio → 1, ImageSize → 200]
{{0.461905, 0.593857, 0.361905, 0.453947},
{0.395904, 0.52795, 0.351893, 0.461929}, {0.383901, 0.515957, 0.351893, 0.475962},
{0.383901, 0.517949, 0.357895, 0.489933}, {0.389937, 0.523952, 0.361905, 0.503968},
{0.395904, 0.533937, 0.367893, 0.515957}, {0.403922, 0.545894, 0.373874, 0.52795},
{0.411917, 0.555932, 0.379888, 0.539924}, {0.419913, 0.568, 0.385906, 0.549906},
{0.425926, 0.58, 0.389908, 0.559965}, {0.433921, 0.589977, 0.393895, 0.567961},
{0.439904, 0.599828, 0.397906, 0.576037}, {0.445902, 0.609865, 0.401914, 0.584},
{0.451948, 0.617886, 0.405904, 0.59}, {0.457944, 0.627859, 0.407915, 0.59601},
{0.46395, 0.635897, 0.411924, 0.60199}, {0.467925, 0.643939, 0.413934, 0.607955},
{0.473934, 0.649971, 0.415909, 0.613971}, {0.477941, 0.657963, 0.41791, 0.617925},
{0.481982, 0.664042, 0.419913, 0.621891}, {0.485944, 0.672, 0.421911, 0.625935}}

```

```
{0.487923, 0.55597, 0.419913, 0.46595},
{0.617886, 0.708, 0.443925, 0.507979}, {0.609929, 0.689956, 0.437931, 0.493927},
{0.59596, 0.670034, 0.429936, 0.479943}, {0.583969, 0.647959, 0.423913, 0.46595},
{0.57, 0.62987, 0.415909, 0.453947}, {0.555891, 0.609877, 0.40991, 0.441926},
{0.543919, 0.591973, 0.403922, 0.429929}, {0.531943, 0.576, 0.395904, 0.419913},
{0.519969, 0.55988, 0.389908, 0.407915}, {0.50996, 0.543919, 0.383901, 0.397906},
{0.497992, 0.529954, 0.377907, 0.389908}, {0.487864, 0.515957, 0.371875, 0.379888},
{0.477941, 0.501961, 0.365931, 0.371875}, {0.467949, 0.489914, 0.359882, 0.363905},
{0.45993, 0.477941, 0.353909, 0.355882}, {0.449883, 0.46595, 0.347945, 0.347909},
{0.441926, 0.455959, 0.343915, 0.339901}, {0.433898, 0.445902, 0.3379, 0.331878},
{0.425952, 0.435967, 0.331878, 0.325893}, {0.41791, 0.425926, 0.327899, 0.319892}}
```

```
{0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5,
0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5}
```



plot - fecundity

```
f111 =
Table[
GP[1] *

$$\left( \text{GI}[1, 1] + \frac{\text{high}[[i]][[1]]}{\frac{1}{2} (\text{high}[[i]][[1]] + \text{high}[[i]][[1]])} \right. \\ \left. \left( 1 - \frac{1}{2} (\text{high}[[i]][[1]] + \text{high}[[i]][[1]]) \right) \right), \{i, 1, \text{Length}[\text{high}]\}]$$

```

```
f121 =
Table[
GP[1] *
```

$$\left(\text{GI}[1, 1] + \frac{\text{high}[[i]][[2]]}{\frac{1}{2} (\text{high}[[i]][[2]] + \text{high}[[i]][[3]])} \right. \\ \left. \left(1 - \frac{1}{2} (\text{high}[[i]][[2]] + \text{high}[[i]][[3]]) \right) \right), \{i, 1, \text{Length}[\text{high}]\}]$$

f211 =

```
Table[
  GP[1] *
  \left( \text{GI}[2, 1] + \frac{\text{high}[[i]][[3]]}{\frac{1}{2} (\text{high}[[i]][[2]] + \text{high}[[i]][[3]])} \right.
  \left. \left( 1 - \frac{1}{2} (\text{high}[[i]][[2]] + \text{high}[[i]][[3]]) \right) \right), \{i, 1, \text{Length}[\text{high}]\}]
```

f221 =

```
Table[
  GP[1] *
  \left( \text{GI}[2, 1] + \frac{\text{high}[[i]][[4]]}{\frac{1}{2} (\text{high}[[i]][[4]] + \text{high}[[i]][[4]])} \right.
  \left. \left( 1 - \frac{1}{2} (\text{high}[[i]][[4]] + \text{high}[[i]][[4]]) \right) \right), \{i, 1, \text{Length}[\text{high}]\}]
```

f112 =

```
Table[
  GP[2] *
  \left( \text{GI}[1, 2] + \frac{\text{low}[[i]][[1]]}{\frac{1}{2} (\text{low}[[i]][[1]] + \text{low}[[i]][[1]])} \right.
  \left. \left( 1 - \frac{1}{2} (\text{low}[[i]][[1]] + \text{low}[[i]][[1]]) \right) \right), \{i, 1, \text{Length}[\text{low}]\}]
```

f122 =

```
Table[
  GP[2] *
  \left( \text{GI}[1, 2] + \frac{\text{low}[[i]][[2]]}{\frac{1}{2} (\text{low}[[i]][[2]] + \text{low}[[i]][[3]])} \right.
  \left. \left( 1 - \frac{1}{2} (\text{low}[[i]][[2]] + \text{low}[[i]][[3]]) \right) \right), \{i, 1, \text{Length}[\text{low}]\}]
```

f212 =

```
Table[
  GP[2] *
  
$$\left( \text{GI}[2, 2] + \frac{\text{low}[[i]][[3]]}{\frac{1}{2} (\text{low}[[i]][[2]] + \text{low}[[i]][[3]])} \right. \\ \left. \left( 1 - \frac{1}{2} (\text{low}[[i]][[2]] + \text{low}[[i]][[3]]) \right) \right), \{i, 1, \text{Length}[\text{low}]\}]$$

```

```
f222 =
```

```
Table[
  GP[2] *
  
$$\left( \text{GI}[2, 2] + \frac{\text{low}[[i]][[4]]}{\frac{1}{2} (\text{low}[[i]][[4]] + \text{low}[[i]][[4]])} \right. \\ \left. \left( 1 - \frac{1}{2} (\text{low}[[i]][[4]] + \text{low}[[i]][[4]]) \right) \right), \{i, 1, \text{Length}[\text{low}]\}]$$

```

```
fig4C = ListLinePlot[{Reverse[f111], Reverse[f121], Reverse[f211],
  Reverse[f221]}, PlotRange → {{0.5, Length[high] + 0.5}, {0, 2}},
  Frame → True,
  FrameLabel →
    {{Text[Style["fecundity, f", FontSize → 11, FontFamily → "Arial"]],
      None}, {None, None}},
  FrameTicks →
    {{{{0, "0.0", {0, 0.01}}, {0.5, "0.5", {0, 0.01}},
      {1, "1.0", {0, 0.01}}, {1.5, "1.5", {0, 0.01}},
      {2, "2.0", {0, 0.01}}}, None},
    {{{1, "-1.0", {0, 0.01}}, {6, "-0.5", {0, 0.01}},
      {11, "0.0", {0, 0.01}}, {16, "0.5", {0, 0.01}},
      {21, "1.0", {0, 0.01}}}, None}},
  PlotStyle → {Directive[RGBColor[1, 0, 0], Dashed, Thickness[0.001]],
    Directive[RGBColor[.5, 0, 0], Dashed, Thickness[0.001]],
    Directive[RGBColor[0, 0, .5], Dashed, Thickness[0.001]],
    Directive[RGBColor[0, 0, 1], Dashed, Thickness[0.001]]},
  PlotMarkers → {"●", 8}, {"▲", 9}, {"△", 9}, {"○", 8}},
  AspectRatio → 1, ImageSize → 200]
```

```
fig4D = ListLinePlot[{Reverse[f112], Reverse[f122], Reverse[f212],
  Reverse[f222]}, PlotRange → {{0.5, Length[high] + 0.5}, {0, 0.2}},
  Frame → True,
  FrameLabel →
    {{Text[Style["fecundity, f", FontSize → 11, FontFamily → "Arial"]],
      None}, {None, None}},
```

FrameTicks →

```
{{{{0, "0.00", {0, 0.01}}, {0.05, "0.05", {0, 0.01}},
  {0.1, "0.10", {0, 0.01}}, {0.15, "0.15", {0, 0.01}},
  {0.2, "0.20", {0, 0.01}}}}, None},
{{{1, "-1.0", {0, 0.01}}, {6, "-0.5", {0, 0.01}},
  {11, "0.0", {0, 0.01}}, {16, "0.5", {0, 0.01}},
  {21, "1.0", {0, 0.01}}}}, None}},
```

PlotStyle → {Directive[RGBColor[1, 0, 0], Dashed, Thickness[0.001]],

Directive[RGBColor[.5, 0, 0], Dashed, Thickness[0.001]],

Directive[RGBColor[0, 0, .5], Dashed, Thickness[0.001]],

Directive[RGBColor[0, 0, 1], Dashed, Thickness[0.001]]},

PlotMarkers → {"●", 8}, {"▲", 9}, {"△", 9}, {"○", 8}},

AspectRatio → 1, ImageSize → 205]

```
{1.5381, 1.6041, 1.6161, 1.6161, 1.61006, 1.6041, 1.59608,
 1.58808, 1.58009, 1.57407, 1.56608, 1.5601, 1.5541, 1.54805,
 1.54206, 1.53605, 1.53208, 1.52607, 1.52206, 1.51802, 1.51406}

{1.64883, 1.67215, 1.67309, 1.66479, 1.65898, 1.65018, 1.64113,
 1.63219, 1.62289, 1.61599, 1.60932, 1.60255, 1.59567, 1.58917,
 1.58449, 1.57785, 1.57348, 1.56962, 1.56516, 1.56118, 1.55662}

{0.495407, 0.548006, 0.55906, 0.559363, 0.555168, 0.547988, 0.539101,
 0.531995, 0.523201, 0.514102, 0.506809, 0.499713, 0.492556, 0.48704,
 0.479738, 0.474324, 0.468644, 0.464496, 0.458966, 0.454866, 0.44947}

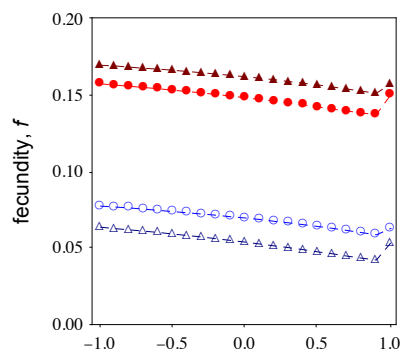
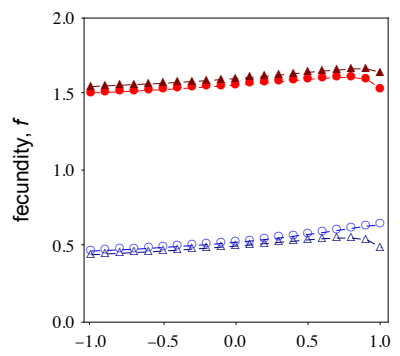
{0.646053, 0.638071, 0.624038, 0.610067, 0.596032, 0.584043,
 0.57205, 0.560076, 0.550094, 0.540035, 0.532039, 0.523963, 0.516, 0.51,
 0.50399, 0.49801, 0.492045, 0.486029, 0.482075, 0.478109, 0.474065}

{0.151208, 0.138211, 0.139007, 0.140404, 0.141603, 0.143, 0.144411,
 0.145608, 0.146806, 0.148003, 0.149004, 0.150201, 0.151214, 0.152206,
 0.153205, 0.154007, 0.155012, 0.155807, 0.15661, 0.157405, 0.158209}

{0.158345, 0.152125, 0.153349, 0.154824, 0.156106, 0.157472, 0.158621,
 0.159685, 0.16093, 0.161908, 0.162855, 0.163752, 0.164633, 0.165478,
 0.16631, 0.167116, 0.167904, 0.168412, 0.169189, 0.169959, 0.170411}

{0.0540668, 0.0426828, 0.043862, 0.0451787, 0.0467064, 0.0479496, 0.0494003,
 0.0507252, 0.0518794, 0.0531134, 0.0543632, 0.0554615, 0.0565839, 0.0577333,
 0.0587103, 0.0596987, 0.0607067, 0.0616009, 0.0624308, 0.0632562, 0.0642061}

{0.063405, 0.0592021, 0.0606073, 0.0620057, 0.063405, 0.0646053, 0.0658074,
 0.0670071, 0.0680087, 0.0692085, 0.0702094, 0.0710092, 0.0720112, 0.0728125,
 0.0736095, 0.0744118, 0.0752091, 0.0760099, 0.0768122, 0.0774107, 0.0780108}
```



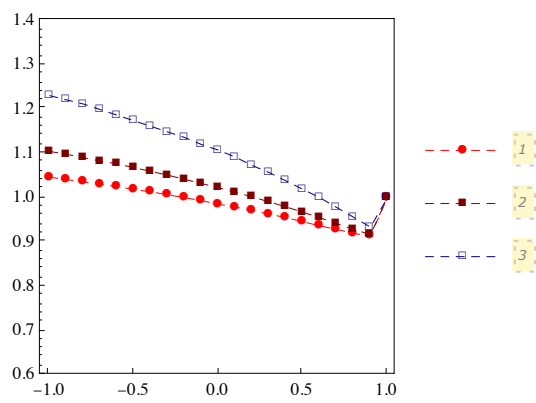
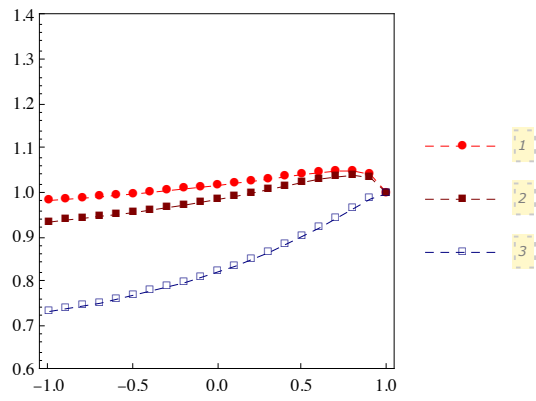
plot - productivity

```
p111 = 2 f111;
p121 = f121 + f211;
p221 = 2 f221;
```

```
p112 = 2 f112;
p122 = f122 + f212;
p222 = 2 f222;
```

```
ListLinePlot[ $\left\{\frac{\text{Reverse}[p111]}{p111[[1]]}, \frac{\text{Reverse}[p121]}{p121[[1]]}, \frac{\text{Reverse}[p221]}{p221[[1]]}\right\}$ ,
  PlotRange → {{0.5, Length[high] + 0.5}, {0.6, 1.4}}, Frame → True,
  FrameTicks → {{Automatic, None},
    {{{1, "-1.0"}, {6, "-0.5"}, {11, "0.0"}, {16, "0.5"}, {21, "1.0"}},
    None}},
  PlotStyle → {Directive[RGBColor[1, 0, 0], Dashed, Thickness[0.001]],
    Directive[RGBColor[.5, 0, 0], Dashed, Thickness[0.001]],
    Directive[RGBColor[0, 0, .5], Dashed, Thickness[0.001]],
    Directive[RGBColor[0, 0, 1], Dashed, Thickness[0.001]]},
  PlotLegends → Automatic,
  PlotMarkers → {"●", 8}, {"■", 8}, {"□", 8}, {"○", 8}},
  AspectRatio → 1, ImageSize → 200]

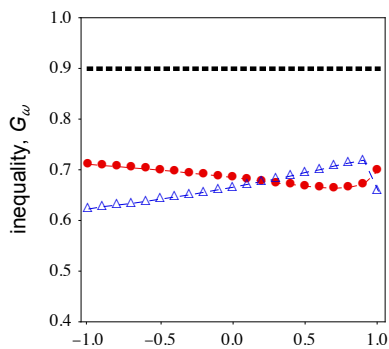
ListLinePlot[ $\left\{\frac{\text{Reverse}[p112]}{p112[[1]]}, \frac{\text{Reverse}[p122]}{p122[[1]]}, \frac{\text{Reverse}[p222]}{p222[[1]]}\right\}$ ,
  PlotRange → {{0.5, Length[high] + 0.5}, {0.6, 1.4}}, Frame → True,
  FrameTicks → {{Automatic, None},
    {{{1, "-1.0"}, {6, "-0.5"}, {11, "0.0"}, {16, "0.5"}, {21, "1.0"}},
    None}},
  PlotStyle → {Directive[RGBColor[1, 0, 0], Dashed, Thickness[0.001]],
    Directive[RGBColor[.5, 0, 0], Dashed, Thickness[0.001]],
    Directive[RGBColor[0, 0, .5], Dashed, Thickness[0.001]],
    Directive[RGBColor[0, 0, 1], Dashed, Thickness[0.001]]},
  PlotLegends → Automatic,
  PlotMarkers → {"●", 8}, {"■", 8}, {"□", 8}, {"○", 8}},
  AspectRatio → 1, ImageSize → 200]
```



plot - inequality

fig5b =

```
ListLinePlot[{{Table[0.9, {i, 1, Length[f121]}], Reverse[1 -  $\frac{f_{211}}{f_{121}}$ ],
  Reverse[1 -  $\frac{f_{212}}{f_{122}}$ ]}, PlotRange -> {{0.5, Length[high] + 0.5}, {0.4, 1}},
Frame -> True,
FrameLabel ->
{{Text[Style["inequality,  $G_\omega$ ", FontSize -> 11, FontFamily -> "Arial"]],
  None}, {None, None}},
FrameTicks ->
{{{0, "0.0", {0, 0.01}}, {0.2, "0.2", {0, 0.01}},
  {0.4, "0.4", {0, 0.01}}, {0.5, "0.5", {0, 0.01}},
  {0.6, "0.6", {0, 0.01}}, {0.7, "0.7", {0, 0.01}},
  {0.8, "0.8", {0, 0.01}}, {0.9, "0.9", {0, 0.01}},
  {1, "1.0", {0, 0.01}}}, None},
{{1, "-1.0", {0, 0.01}}, {6, "-0.5", {0, 0.01}},
  {11, "0.0", {0, 0.01}}, {16, "0.5", {0, 0.01}},
  {21, "1.0", {0, 0.01}}}, None}},
PlotStyle -> {Directive[RGBColor[0, 0, 0], Dotted, Thickness[0.015]],
  Directive[RGBColor[.9, 0, 0], Dashed, Thickness[0.001]],
  Directive[RGBColor[0, 0, .9], Dashed, Thickness[0.001]],
  Directive[RGBColor[0, 0, 1], Dashed, Thickness[0.001]]},
PlotMarkers -> {{{"", 8}, {"●", 8}, {"△", 10}, {"○", 8}},
AspectRatio -> 1, ImageSize -> 200]
```



```

GB = 1 - 
$$\frac{p[2] (q[1, 1] p112 + q[1, 2] p122 + q[2, 2] p222)}{p[1] (q[1, 1] p111 + q[1, 2] p121 + q[2, 2] p221)}$$

fig5c = ListLinePlot[{{Table[1 -  $\frac{1}{10}$ , {i, 1, Length[GB]}]}, Reverse[GB]}},
  PlotRange → {{0.5, Length[high] + 0.5}, {0.8, 1}}, Frame → True,
  FrameLabel →
    {{Text[Style["inequality,  $G_\beta$ ", FontSize → 11, FontFamily → "Arial"]],
      None}, {None, None}},
  FrameTicks →
    {{{{0, "0.0", {0, 0.01}}, {0.2, "0.2", {0, 0.01}},
      {0.4, "0.4", {0, 0.01}}, {0.5, "0.5", {0, 0.01}},
      {0.6, "0.6", {0, 0.01}}, {0.7, "0.7", {0, 0.01}},
      {0.8, "0.8", {0, 0.01}}, {0.9, "0.9", {0, 0.01}},
      {1, "1.0", {0, 0.01}}}, None},
    {{{1, "-1.0", {0, 0.01}}, {6, "-0.5", {0, 0.01}},
      {11, "0.0", {0, 0.01}}, {16, "0.5", {0, 0.01}},
      {21, "1.0", {0, 0.01}}}, None}},
  PlotStyle → {Directive[RGBColor[0, 0, 0], Dotted, Thickness[0.015]],
    Directive[RGBColor[.9, 0, 0], Dashed, Thickness[0.001]],
    Directive[RGBColor[0, 0, .9], Dashed, Thickness[0.001]],
    Directive[RGBColor[0, 0, 1], Dashed, Thickness[0.001]]},
  PlotMarkers → {{{"", 8}, {"●", 8}, {"△", 10}, {"○", 8}},
  AspectRatio → 1, ImageSize → 200]
{0.901343, 0.912104, 0.91127, 0.909577, 0.907738, 0.905837, 0.903817,
0.901903, 0.900001, 0.898159, 0.896439, 0.894795, 0.89312, 0.891582,
0.890072, 0.888593, 0.887161, 0.885861, 0.884555, 0.883347, 0.88212}

```

