

**/* Voltage drop across the myelin sheath of an myelinated axon,
under a transverse electric field exposure */**



/ * Step 1 : voltage distributatio inside the myelin sheath. * /

$$\begin{aligned}
 V1 = & - \left(\left(2 a^2 E0 S0 \left(c^2 r^2 (S1 - S2) \left(d^2 (S2 + S3) (S3 - S4) + c^2 (S2 - S3) (S3 + S4) \right) + \right. \right. \right. \\
 & b^4 (S1 - S2) \left(d^2 (S2 - S3) (S3 - S4) + c^2 (S2 + S3) (S3 + S4) \right) + \\
 & b^2 (S1 + S2) \left(d^2 r^2 (S2 - S3) (S3 - S4) + c^4 (S2 - S3) (S3 + S4) + \right. \\
 & \left. \left. c^2 (S2 + S3) \left(d^2 (S3 - S4) + r^2 (S3 + S4) \right) \right) \right) \text{Cos}[\theta] \right) / \\
 & \left(r \left(b^2 (S0 - S1) \left(c^2 (S1 + S2) \left(d^2 (S2 + S3) (S3 - S4) + c^2 (S2 - S3) (S3 + S4) \right) + \right. \right. \right. \\
 & b^2 (S1 - S2) \left(d^2 (S2 - S3) (S3 - S4) + c^2 (S2 + S3) (S3 + S4) \right) \right) + \\
 & a^2 (S0 + S1) \left(c^2 (S1 - S2) \left(d^2 (S2 + S3) (S3 - S4) + c^2 (S2 - S3) (S3 + S4) \right) + \right. \\
 & \left. \left. b^2 (S1 + S2) \left(d^2 (S2 - S3) (S3 - S4) + c^2 (S2 + S3) (S3 + S4) \right) \right) \right) \right) \\
 - & \left(\left(2 a^2 E0 S0 \left(c^2 r^2 (S1 - S2) \left(d^2 (S2 + S3) (S3 - S4) + c^2 (S2 - S3) (S3 + S4) \right) + \right. \right. \right. \\
 & b^4 (S1 - S2) \left(d^2 (S2 - S3) (S3 - S4) + c^2 (S2 + S3) (S3 + S4) \right) + \\
 & b^2 (S1 + S2) \left(d^2 r^2 (S2 - S3) (S3 - S4) + c^4 (S2 - S3) (S3 + S4) + \right. \\
 & \left. \left. c^2 (S2 + S3) \left(d^2 (S3 - S4) + r^2 (S3 + S4) \right) \right) \right) \text{Cos}[\theta] \right) / \\
 & \left(r \left(b^2 (S0 - S1) \left(c^2 (S1 + S2) \left(d^2 (S2 + S3) (S3 - S4) + c^2 (S2 - S3) (S3 + S4) \right) + \right. \right. \right. \\
 & b^2 (S1 - S2) \left(d^2 (S2 - S3) (S3 - S4) + c^2 (S2 + S3) (S3 + S4) \right) \right) + \\
 & a^2 (S0 + S1) \left(c^2 (S1 - S2) \left(d^2 (S2 + S3) (S3 - S4) + c^2 (S2 - S3) (S3 + S4) \right) + \right. \\
 & \left. \left. b^2 (S1 + S2) \left(d^2 (S2 - S3) (S3 - S4) + c^2 (S2 + S3) (S3 + S4) \right) \right) \right) \right)
 \end{aligned}$$

r = a

a

Vlout =

$$\begin{aligned}
 - & \left(\left(2 a^2 E0 S0 \left(c^2 r^2 (S1 - S2) \left(d^2 (S2 + S3) (S3 - S4) + c^2 (S2 - S3) (S3 + S4) \right) + b^4 (S1 - S2) \right. \right. \right. \\
 & \left(d^2 (S2 - S3) (S3 - S4) + c^2 (S2 + S3) (S3 + S4) \right) + \\
 & b^2 (S1 + S2) \left(d^2 r^2 (S2 - S3) (S3 - S4) + c^4 (S2 - S3) (S3 + S4) + \right. \\
 & \left. \left. c^2 (S2 + S3) \left(d^2 (S3 - S4) + r^2 (S3 + S4) \right) \right) \right) \text{Cos}[\theta] \right) / \\
 & \left(r \left(b^2 (S0 - S1) \left(c^2 (S1 + S2) \left(d^2 (S2 + S3) (S3 - S4) + c^2 (S2 - S3) (S3 + S4) \right) + \right. \right. \right. \\
 & b^2 (S1 - S2) \left(d^2 (S2 - S3) (S3 - S4) + c^2 (S2 + S3) (S3 + S4) \right) \right) + \\
 & a^2 (S0 + S1) \left(c^2 (S1 - S2) \left(d^2 (S2 + S3) (S3 - S4) + c^2 (S2 - S3) (S3 + S4) \right) + \right. \\
 & \left. \left. b^2 (S1 + S2) \left(d^2 (S2 - S3) (S3 - S4) + c^2 (S2 + S3) (S3 + S4) \right) \right) \right) \right) \\
 - & \left(\left(2 a E0 S0 \left(a^2 c^2 (S1 - S2) \left(d^2 (S2 + S3) (S3 - S4) + c^2 (S2 - S3) (S3 + S4) \right) + \right. \right. \right. \\
 & b^4 (S1 - S2) \left(d^2 (S2 - S3) (S3 - S4) + c^2 (S2 + S3) (S3 + S4) \right) + \\
 & b^2 (S1 + S2) \left(a^2 d^2 (S2 - S3) (S3 - S4) + c^4 (S2 - S3) (S3 + S4) + \right. \\
 & \left. \left. c^2 (S2 + S3) \left(d^2 (S3 - S4) + a^2 (S3 + S4) \right) \right) \right) \text{Cos}[\theta] \right) / \\
 & \left(b^2 (S0 - S1) \left(c^2 (S1 + S2) \left(d^2 (S2 + S3) (S3 - S4) + c^2 (S2 - S3) (S3 + S4) \right) + \right. \right. \\
 & b^2 (S1 - S2) \left(d^2 (S2 - S3) (S3 - S4) + c^2 (S2 + S3) (S3 + S4) \right) \right) + \\
 & a^2 (S0 + S1) \left(c^2 (S1 - S2) \left(d^2 (S2 + S3) (S3 - S4) + c^2 (S2 - S3) (S3 + S4) \right) + \right. \\
 & \left. \left. b^2 (S1 + S2) \left(d^2 (S2 - S3) (S3 - S4) + c^2 (S2 + S3) (S3 + S4) \right) \right) \right)
 \end{aligned}$$

$$r = b$$

$$b$$

$$V_{lin} =$$

$$\begin{aligned}
& - \left(\left(2 a^2 E_0 S_0 \left(c^2 r^2 (S_1 - S_2) \left(d^2 (S_2 + S_3) (S_3 - S_4) + c^2 (S_2 - S_3) (S_3 + S_4) \right) + b^4 (S_1 - S_2) \right. \right. \right. \\
& \quad \left. \left. \left(d^2 (S_2 - S_3) (S_3 - S_4) + c^2 (S_2 + S_3) (S_3 + S_4) \right) + \right. \right. \\
& \quad \left. \left. b^2 (S_1 + S_2) \left(d^2 r^2 (S_2 - S_3) (S_3 - S_4) + c^4 (S_2 - S_3) (S_3 + S_4) + \right. \right. \right. \\
& \quad \left. \left. \left. c^2 (S_2 + S_3) \left(d^2 (S_3 - S_4) + r^2 (S_3 + S_4) \right) \right) \right) \right) \text{Cos}[\theta] \right) / \\
& \left(r \left(b^2 (S_0 - S_1) \left(c^2 (S_1 + S_2) \left(d^2 (S_2 + S_3) (S_3 - S_4) + c^2 (S_2 - S_3) (S_3 + S_4) \right) + \right. \right. \right. \\
& \quad \left. \left. b^2 (S_1 - S_2) \left(d^2 (S_2 - S_3) (S_3 - S_4) + c^2 (S_2 + S_3) (S_3 + S_4) \right) \right) + \right. \\
& \quad \left. a^2 (S_0 + S_1) \left(c^2 (S_1 - S_2) \left(d^2 (S_2 + S_3) (S_3 - S_4) + c^2 (S_2 - S_3) (S_3 + S_4) \right) + \right. \right. \\
& \quad \left. \left. b^2 (S_1 + S_2) \left(d^2 (S_2 - S_3) (S_3 - S_4) + c^2 (S_2 + S_3) (S_3 + S_4) \right) \right) \right) \right) \\
& - \left(\left(2 a^2 E_0 S_0 \left(b^2 c^2 (S_1 - S_2) \left(d^2 (S_2 + S_3) (S_3 - S_4) + c^2 (S_2 - S_3) (S_3 + S_4) \right) + \right. \right. \right. \\
& \quad \left. \left. b^4 (S_1 - S_2) \left(d^2 (S_2 - S_3) (S_3 - S_4) + c^2 (S_2 + S_3) (S_3 + S_4) \right) + \right. \right. \\
& \quad \left. \left. b^2 (S_1 + S_2) \left(b^2 d^2 (S_2 - S_3) (S_3 - S_4) + c^4 (S_2 - S_3) (S_3 + S_4) + \right. \right. \right. \\
& \quad \left. \left. \left. c^2 (S_2 + S_3) \left(d^2 (S_3 - S_4) + b^2 (S_3 + S_4) \right) \right) \right) \right) \text{Cos}[\theta] \right) / \\
& \left(b \left(b^2 (S_0 - S_1) \left(c^2 (S_1 + S_2) \left(d^2 (S_2 + S_3) (S_3 - S_4) + c^2 (S_2 - S_3) (S_3 + S_4) \right) + \right. \right. \right. \\
& \quad \left. \left. b^2 (S_1 - S_2) \left(d^2 (S_2 - S_3) (S_3 - S_4) + c^2 (S_2 + S_3) (S_3 + S_4) \right) \right) + \right. \\
& \quad \left. a^2 (S_0 + S_1) \left(c^2 (S_1 - S_2) \left(d^2 (S_2 + S_3) (S_3 - S_4) + c^2 (S_2 - S_3) (S_3 + S_4) \right) + \right. \right. \\
& \quad \left. \left. b^2 (S_1 + S_2) \left(d^2 (S_2 - S_3) (S_3 - S_4) + c^2 (S_2 + S_3) (S_3 + S_4) \right) \right) \right) \right)
\end{aligned}$$

/ Step 2 : Voltage drop across the myelin sheath is

defined as the voltage difference between the two sides. * /

$$V_{myelin} = V_{lin} - V_{lout}$$

$$\begin{aligned}
& \left(2 a E_0 S_0 \left(a^2 c^2 (S_1 - S_2) \left(d^2 (S_2 + S_3) (S_3 - S_4) + c^2 (S_2 - S_3) (S_3 + S_4) \right) + \right. \right. \\
& \quad \left. \left. b^4 (S_1 - S_2) \left(d^2 (S_2 - S_3) (S_3 - S_4) + c^2 (S_2 + S_3) (S_3 + S_4) \right) + \right. \right. \\
& \quad \left. \left. b^2 (S_1 + S_2) \left(a^2 d^2 (S_2 - S_3) (S_3 - S_4) + c^4 (S_2 - S_3) (S_3 + S_4) + \right. \right. \right. \\
& \quad \left. \left. \left. c^2 (S_2 + S_3) \left(d^2 (S_3 - S_4) + a^2 (S_3 + S_4) \right) \right) \right) \right) \text{Cos}[\theta] \right) / \\
& \left(b^2 (S_0 - S_1) \left(c^2 (S_1 + S_2) \left(d^2 (S_2 + S_3) (S_3 - S_4) + c^2 (S_2 - S_3) (S_3 + S_4) \right) + \right. \right. \\
& \quad \left. \left. b^2 (S_1 - S_2) \left(d^2 (S_2 - S_3) (S_3 - S_4) + c^2 (S_2 + S_3) (S_3 + S_4) \right) \right) + \right. \\
& \quad \left. a^2 (S_0 + S_1) \left(c^2 (S_1 - S_2) \left(d^2 (S_2 + S_3) (S_3 - S_4) + c^2 (S_2 - S_3) (S_3 + S_4) \right) + \right. \right. \\
& \quad \left. \left. b^2 (S_1 + S_2) \left(d^2 (S_2 - S_3) (S_3 - S_4) + c^2 (S_2 + S_3) (S_3 + S_4) \right) \right) \right) \right) - \\
& \left(2 a^2 E_0 S_0 \left(b^2 c^2 (S_1 - S_2) \left(d^2 (S_2 + S_3) (S_3 - S_4) + c^2 (S_2 - S_3) (S_3 + S_4) \right) + \right. \right. \\
& \quad \left. \left. b^4 (S_1 - S_2) \left(d^2 (S_2 - S_3) (S_3 - S_4) + c^2 (S_2 + S_3) (S_3 + S_4) \right) + \right. \right. \\
& \quad \left. \left. b^2 (S_1 + S_2) \left(b^2 d^2 (S_2 - S_3) (S_3 - S_4) + c^4 (S_2 - S_3) (S_3 + S_4) + \right. \right. \right. \\
& \quad \left. \left. \left. c^2 (S_2 + S_3) \left(d^2 (S_3 - S_4) + b^2 (S_3 + S_4) \right) \right) \right) \right) \text{Cos}[\theta] \right) / \\
& \left(b \left(b^2 (S_0 - S_1) \left(c^2 (S_1 + S_2) \left(d^2 (S_2 + S_3) (S_3 - S_4) + c^2 (S_2 - S_3) (S_3 + S_4) \right) + \right. \right. \right. \\
& \quad \left. \left. b^2 (S_1 - S_2) \left(d^2 (S_2 - S_3) (S_3 - S_4) + c^2 (S_2 + S_3) (S_3 + S_4) \right) \right) + \right. \\
& \quad \left. a^2 (S_0 + S_1) \left(c^2 (S_1 - S_2) \left(d^2 (S_2 + S_3) (S_3 - S_4) + c^2 (S_2 - S_3) (S_3 + S_4) \right) + \right. \right. \\
& \quad \left. \left. b^2 (S_1 + S_2) \left(d^2 (S_2 - S_3) (S_3 - S_4) + c^2 (S_2 + S_3) (S_3 + S_4) \right) \right) \right) \right)
\end{aligned}$$

FullSimplify[%6]

$$\begin{aligned} & (2 a (a - b) E_0 S_0 (b (c^2 (S_1 + S_2) (d^2 (S_2 + S_3) (-S_3 + S_4) - c^2 (S_2 - S_3) (S_3 + S_4)) - \\ & \quad b^2 (S_1 - S_2) (d^2 (S_2 - S_3) (S_3 - S_4) + c^2 (S_2 + S_3) (S_3 + S_4))) + \\ & \quad a (c^2 (S_1 - S_2) (d^2 (S_2 + S_3) (S_3 - S_4) + c^2 (S_2 - S_3) (S_3 + S_4)) + \\ & \quad \quad b^2 (S_1 + S_2) (d^2 (S_2 - S_3) (S_3 - S_4) + c^2 (S_2 + S_3) (S_3 + S_4))) \cos[\theta]) / \\ & (b^2 (S_0 - S_1) (c^2 (S_1 + S_2) (d^2 (S_2 + S_3) (S_3 - S_4) + c^2 (S_2 - S_3) (S_3 + S_4)) + \\ & \quad b^2 (S_1 - S_2) (d^2 (S_2 - S_3) (S_3 - S_4) + c^2 (S_2 + S_3) (S_3 + S_4))) + \\ & \quad a^2 (S_0 + S_1) (c^2 (S_1 - S_2) (d^2 (S_2 + S_3) (S_3 - S_4) + c^2 (S_2 - S_3) (S_3 + S_4)) + \\ & \quad \quad b^2 (S_1 + S_2) (d^2 (S_2 - S_3) (S_3 - S_4) + c^2 (S_2 + S_3) (S_3 + S_4)))) \end{aligned}$$

**/ * Step 3 : Validation : if myelin thickness is zero,
there shall be no voltage drop across it. * /**



a = b

b

Vmyelin

0