SUPPORTING INFORMATION

Hanley. B., P. Connelly, and B. Dennis. 201X. Another look at the eigenvalues of a population matrix model.

**Mathematical derivations**

Given a population matrix representing any 3-stage life history, where the vital rates are arbitrarily denoted for , and , and of the form:

, (1S)

then the characteristic equation is in the form (Beyer, 1978):

, (2S)

where is the identity matrix, and is an eigenvalue of . Equivalently,

, (3S)

where

, (4S)

), (5S)

and

). (6S)

Werefer toand from eqs. 4S-6S as superparameters (e.g. Hanley & Dennis, 2019). The roots of the characteristic equation are the eigenvalues of the matrix (Cull & Vogt, 1973).

Let be a real number. Rewriting, eq. 3S, we have:

(7S)

Collecting positive terms on one side and negative terms on the other, we have:

, (8S)

which leads to positive and negative contributions of the balance equation (see Table 1).

Now let be a complex number, where The characteristic equation for a 3-stage population matrix model is then:

, (9S)

Where p, q, and are as in eqs. 4S-6S. Rewriting eqs. 3S, we have:

(10S)

Expanding, we have:

(11S)

Further:

(12S)

Collecting positive terms on one side and negative terms on the other, we have:

. (13S)

Collecting real and imaginary components, we have:

+(

=( )+( . (14S)

This implies that:

, (15S)

and

. (16S)

Equations 15S-16S leads to two sets of positive and negative contributions of the balance equation (see Tables S.1 and S.2).

|  |  |
| --- | --- |
| Components of the characteristic balance equation for an  imaginary dominant eigenvalue | |
| Negative contribution for the real component () | Positive contribution for the real component () |
|  |  |
|  |  |
|  |  |
| ) |  |

Table S.1. The positive and negative contributions to the real component of the balance equation for a complex dominant eigenvalue. Here, is the term that must vary to balance these components to zero.

|  |  |
| --- | --- |
| Components of the characteristic balance equation for an  imaginary dominant eigenvalue | |
| Negative contribution for the imaginary component (*h*) | Positive contribution for the imaginary component () |
|  |  |
|  | ) |
|  |  |

Table. S.2. The positive and negative contributions to the imaginary component of the balance equation for a complex dominant eigenvalue. Here, is the term that must vary to balance these components to zero.