# Supplementary 1 – Description of the new *flexit* model.

## From logistic to flexit model

The lack of an ideal sigmoid model to describe TSD patterns (i.e., asymmetrical in the transitions toward lower and upper asymptotes) prompted us to develop a new, more versatile sigmoid function, based on the logistic law:

This formula has the advantage that and . Thus, the temperature is the temperature at which 50% of the embryos are males or females. However, this model assumes a symmetric transition around . The *A-logistic* model is an asymmetric sigmoid model, with parameter being a parameter controlling the asymmetry (Godfrey et al. 2003):

As for a logistic model, . When , the transitions from to the asymptotes showed more acute angles whereas when , the transitions from to asymptotes showed more obtuse angles, as compared to logistic model on both sides of . Hulin et al. (2009) observed that the *A-logistic* model requires that both transitions are either acute, or obtuse and that it was not possible to mix both conditions on each side of . We propose here a new approach to alleviate this constraint.

The first-order derivative of the *A-logistic* model is:

With

It follows that slope at depends on both and . As expected, when , .

Then different transitions toward the asymptotes below and above can be defined with for and for .

A smooth transition at requires the same , regardless of the values of and . Then, we search for and values (respectively for and ) that ensure that is equal according to and . It follows that:

and

Being symmetric, a logistic law can be written in two ways:

which does not apply for the A-logistic model as

However, both of these forms are interesting as the influence of on the acute or obtuse transitions toward the asymptotes is reversed. When , the transition toward the asymptote is acute when and obtuse when for the form . However, it becomes acute when and obtuse when for the form .

This property was used to define the flexible-logistic model or *flexit* model:

It should be noted that is always different from 0, and , and .

*A flexit* model uses 4 parameters and a *logistic* model is nested within it. When , the *flexit* model is a *logistic* model with 2 parameters. The model is not defined for or . If such a situation occurs, is replaced by

The *flexit* model is included as a function in the ***HelpersMG*** R package (version 3.7 and higher) (Girondot 2019b) and is included in the tsd() function of the ***embryogrowth*** R package (version 7.5 and higher) (Girondot 2019a).

## Transitional range of temperature (*TRT*) of a *flexit* model of TSD pattern

*TRT l%* is defined as *TRTH*- *TRTL* with *TRTH* being the temperature at which *l* sex ratio is obtained and *TRTL* being the temperature at which 1-*l* sex ratio is obtained according to the definition of Girondot (1999).

When with

When with

It follows that