# Supplementary 1 – Description of the new *flexit* model.

## From logistic to flexit model

The lack of an ideal sigmoid model to describe TSD patterns (i.e., asymmetrical in the transitions toward lower and upper asymptotes) prompted us to develop a new, more versatile sigmoid function, based on the logistic law: $f\left(x\right)=\left(1+e^{4 S \left(P-x\right)}\right)^{-1}$

This formula has the advantage that $f\left(x=P\right)=0.5$ and $f^{'}\left(x=P\right)=S$. Thus, the temperature $P$ is the temperature at which 50% of the embryos are males or females. However, this model assumes a symmetric transition around $P$. The *A-logistic* model is an asymmetric sigmoid model, with parameter $K$ being a parameter controlling the asymmetry (Godfrey et al. 2003):

$$f\left(x\right)=\left(1+\left(2^{K}-1\right)e^{4 S \left(P-x\right)}\right)^{{-1}/{K}}$$

As for a logistic model, $f\left(x=P\right)=0.5$. When $K<1$, the transitions from $P$ to the asymptotes showed more acute angles whereas when $K>1$, the transitions from $P$ to asymptotes showed more obtuse angles, as compared to logistic model on both sides of $P$. Hulin et al. (2009) observed that the *A-logistic* model requires that both transitions are either acute, or obtuse and that it was not possible to mix both conditions on each side of $P$. We propose here a new approach to alleviate this constraint.

The first-order derivative of the *A-logistic* model is:

$$f^{'}\left(x\right)=S \frac{4}{K} \left(2^{K}-1\right) e^{4 S \left(P-x\right)}\left(1+\left(2^{K}-1\right)e^{4 S \left(P-x\right)}\right)^{-\frac{1}{K}-1}$$

With $f^{'}\left(x=P\right)=S\frac{4}{K}\left(2^{K}-1\right)\left(2^{-K}\right)^{\frac{1}{K}+1}$

It follows that slope at $x=P$ depends on both $S$ and $K$. As expected, when $K=1$, $f^{'}\left(x=P\right)=S$.

Then different transitions toward the asymptotes below and above $P$ can be defined with $K=K\_{1}$ for $x<P$ and $K=K\_{2}$ for $x\geq P$.

A smooth transition at $x=P$ requires the same $f^{'}\left(x=P\right)$, regardless of the values of $K\_{1}$ and $K\_{2}$. Then, we search for $S\_{1}$ and $S\_{2}$ values (respectively for $x<P$ and $x\geq P$) that ensure that $f^{'}\left(x=P\right)$ is equal according to $K\_{1}$ and $K\_{2}$. It follows that:

$S\_{1}=f^{'}\left(x=P\right)\frac{\left(2^{-K\_{1}}\right)^{{-1}/{K\_{1}}-1} K\_{1}}{4 \left(2^{K\_{1}}-1\right)}$ and $S\_{2}=f^{'}\left(x=P\right)\frac{\left(2^{-K\_{2}}\right)^{{-1}/{K\_{2}}-1} K\_{2}}{4 \left(2^{K\_{2}}-1\right)}$

Being symmetric, a logistic law can be written in two ways:

$$f\left(x\right)=\left(1+e^{4 S \left(P-x\right)}\right)^{-1}=1-\left(1+e^{4 S \left(x-P\right)}\right)^{-1}$$

which does not apply for the A-logistic model $f\left(x\right)=\left(1+\left(2^{K}-1\right)e^{4 S \left(P-x\right)}\right)^{{-1}/{K}}$ as

$$\left(1+\left(2^{K}-1\right)e^{4 S \left(P-x\right)}\right)^{{-1}/{K}}\ne 1-\left(1+\left(2^{K}-1\right)e^{4 S \left(x-P\right)}\right)^{{-1}/{K}}$$

However, both of these forms are interesting as the influence of $K$ on the acute or obtuse transitions toward the asymptotes is reversed. When $K>1$, the transition toward the asymptote is acute when $x<P$ and obtuse when $x>P$ for the form $\left(1+\left(2^{K}-1\right)e^{4 S \left(P-x\right)}\right)^{{-1}/{K}}$. However, it becomes acute when $x>P$ and obtuse when $x<P$ for the form $1-\left(1+\left(2^{K}-1\right)e^{4 S \left(x-P\right)}\right)^{{-1}/{K}}$.

This property was used to define the flexible-logistic model or *flexit* model:

$$\left\{\begin{matrix}x<P&S\_{1}=\frac{2^{K\_{1}-1} S K\_{1}}{2^{K\_{1}}-1}&f\left(x\right)=\left(1+\left(2^{K\_{1}}-1\right)e^{4 S\_{1} \left(P-x\right)}\right)^{{-1}/{K\_{1}}}\\x\geq P&S\_{2}=\frac{2^{K\_{2}-1} S K\_{2}}{2^{K\_{2}}-1}&f\left(x\right)=1-\left(1+\left(2^{K\_{2}}-1\right)e^{4 S\_{2} \left(x-P\right)}\right)^{{-1}/{K\_{2}}}\end{matrix}\right.$$

It should be noted that $2^{K\_{i}}-1$ is always different from 0, and $\lim\_{K\_{i}\to -\infty }S\_{i}=0$, and $\lim\_{K\_{i}\to +\infty }S\_{i}=S\_{i}\infty $.

*A flexit* model uses 4 parameters and a *logistic* model is nested within it. When $K\_{1}=K\_{2}=1$, the *flexit* model is a *logistic* model with 2 parameters. The model is not defined for $K\_{1}=0$ or $K\_{2}=0$. If such a situation occurs, $K\_{x}$ is replaced by $10^{-9}.$

The *flexit* model is included as a function in the ***HelpersMG*** R package (version 3.7 and higher) (Girondot 2019b) and is included in the tsd() function of the ***embryogrowth*** R package (version 7.5 and higher) (Girondot 2019a).

## Transitional range of temperature (*TRT*) of a *flexit* model of TSD pattern

*TRT l%* is defined as *TRTH*- *TRTL* with *TRTH* being the temperature at which *l* sex ratio is obtained and *TRTL* being the temperature at which 1-*l* sex ratio is obtained according to the definition of Girondot (1999).

When $x<P then$ $f\left(x\right)=\left(1+\left(2^{K\_{1}}-1\right)e^{4 S\_{1} \left(P-x\right)}\right)^{{-1}/{K\_{1}}}$ with $S\_{1}=\frac{2^{K\_{1}-1} S K\_{1}}{2^{K\_{1}}-1}$

$$\frac{1}{\left(1+\left(2^{K\_{1}}-1\right)e^{4 S\_{1} \left(P-TRT\_{L}\right)}\right)^{{1}/{K\_{1}}}}=1-l$$

$$\left(1+\left(2^{K\_{1}}-1\right)e^{4 S\_{1} \left(P-TRT\_{L}\right)}\right)^{{1}/{K\_{1}}}={1}/{\left(1-l\right)}$$

$$1+\left(2^{K\_{1}}-1\right)e^{4 S\_{1} \left(P-TRT\_{L}\right)}=\left({1}/{\left(1-l\right)}\right)^{K\_{1}}$$

$$\left(2^{K\_{1}}-1\right)e^{4 S\_{1} \left(P-TRT\_{L}\right)}=\left({1}/{\left(1-l\right)}\right)^{K\_{1}}-1$$

$$e^{4 S\_{1} \left(P-TRT\_{L}\right)}=\frac{\left({1}/{\left(1-l\right)}\right)^{K\_{1}}-1}{2^{K\_{1}}-1}$$

$$4 S\_{1} \left(P-TRT\_{L}\right)=ln\frac{\left({1}/{\left(1-l\right)}\right)^{K\_{1}}-1}{2^{K\_{1}}-1}$$

$$P-TRT\_{L}=\frac{1}{4 S\_{1}}ln\frac{\left({1}/{\left(1-l\right)}\right)^{K\_{1}}-1}{2^{K\_{1}}-1}$$

$$TRT\_{L}=P-\frac{1}{4 S\_{1}}ln\frac{\left({1}/{\left(1-l\right)}\right)^{K\_{1}}-1}{2^{K\_{1}}-1}$$

When $x\geq P then$ $f\left(x\right)=1-\left(1+\left(2^{K\_{2}}-1\right)e^{4 S\_{2} \left(TRT\_{H}-P\right)}\right)^{{-1}/{K\_{2}}}$ with $S\_{2}=\frac{2^{K\_{2}-1} S K\_{2}}{2^{K\_{2}}-1}$

$$1-\frac{1}{\left(1+\left(2^{K\_{2}}-1\right)e^{4 S\_{2} \left(TRT\_{H}-P\right)}\right)^{{1}/{K\_{2}}}}=l$$

$$\frac{1}{\left(1+\left(2^{K\_{2}}-1\right)e^{4 S\_{2} \left(TRT\_{H}-P\right)}\right)^{{1}/{K\_{2}}}}=\left(1-l\right)$$

$$\left(1+\left(2^{K\_{2}}-1\right)e^{4 S\_{2} \left(TRT\_{H}-P\right)}\right)^{{1}/{K\_{2}}}={1}/{\left(1-l\right)}$$

$$1+\left(2^{K\_{2}}-1\right)e^{4 S\_{2} \left(TRT\_{H}-P\right)}=\left({1}/{\left(1-l\right)}\right)^{K\_{2}}$$

$$\left(2^{K\_{2}}-1\right)e^{4 S\_{2} \left(TRT\_{H}-P\right)}=\left({1}/{\left(1-l\right)}\right)^{K\_{2}}-1$$

$$e^{4 S\_{2} \left(TRT\_{H}-P\right)}=\frac{\left({1}/{\left(1-l\right)}\right)^{K\_{2}}-1}{2^{K\_{2}}-1}$$

$$4 S\_{2} \left(TRT\_{H}-P\right)=ln\frac{\left({1}/{\left(1-l\right)}\right)^{K\_{2}}-1}{2^{K\_{2}}-1}$$

$$TRT\_{H}-P=\frac{1}{4 S\_{2}}ln\frac{\left({1}/{\left(1-l\right)}\right)^{K\_{2}}-1}{2^{K\_{2}}-1}$$

$$TRT\_{H}=P+\frac{1}{4 S\_{2}}ln\frac{\left({1}/{\left(1-l\right)}\right)^{K\_{2}}-1}{2^{K\_{2}}-1}$$

It follows that $TRT=TRT\_{H}-TRT\_{L}$

$$TRT=P+\frac{1}{4 S\_{2}}ln\frac{\left({1}/{\left(1-l\right)}\right)^{K\_{2}}-1}{2^{K\_{2}}-1}-P+\frac{1}{4 S\_{1}}ln\frac{\left({1}/{\left(1-l\right)}\right)^{K\_{1}}-1}{2^{K\_{1}}-1}$$

$$TRT=\frac{1}{4 S\_{2}}ln\frac{\left({1}/{\left(1-l\right)}\right)^{K\_{2}}-1}{2^{K\_{2}}-1}+\frac{1}{4 S\_{1}}ln\frac{\left({1}/{\left(1-l\right)}\right)^{K\_{1}}-1}{2^{K\_{1}}-1}$$