Appendix 3: $R^2_{adj}$ is a special case of $R^2_{KL,adj}$ for Gaussian distribution

For a Gaussian distribution, the scaled deviances of the fitted and intercept-only models are:

\[
D^*(y, \mu) = \frac{\sum(y_i - \mu_i)^2}{\sigma^2} = \frac{SS_E}{\sigma^2} \tag{1}
\]

\[
D^*(y, \mu^0) = \frac{\sum(y_i - \bar{y})^2}{\sigma^2} = \frac{SS_T}{\sigma^2}
\]

Since $\sigma^2$ is unknown, its unbiased estimate is used in the formula:

\[
\hat{\sigma}^2 = \frac{SS_E}{n - k - 1} \tag{2}
\]

The adjusted Kullback-Leibler $R^2$ is:

\[
R^2_{KL,adj} = \frac{SS_T}{\hat{\sigma}^2} - \frac{SS_E - k}{\hat{\sigma}^2} = \frac{SS_T - SS_E - \hat{\sigma}^2 k}{SS_T} \tag{3}
\]

Substituting (2) into (3), we obtain the commonly used formula for adjusted R-squared:

\[
R^2_{KL,adj} = 1 - \frac{SS_E}{SS_T} \left(1 + \frac{k}{n - k - 1}\right) = 1 - (1 - R^2) \frac{n - 1}{n - k - 1} \tag{4}
\]