**APPENDIX C**

$Correctness Proof of Proposed Algorithm 1$**:** $EnQPBEA-MPM$

***Trace Note – C1:*** *The mathematical trace steps specified in Eqs. (17) – (23) justifies quantum operations for each search pattern* $P\_{k}\in P$*such that all* $t\_{k}$*indices corresponding to*$P\_{k}$ *are identified separately using* $c^{th}$ *quantum core* $QCore\_{c}$ *accessing* $T$ *on shared*$ QMEM.$

1. *Prepare quantum registers of each*$c^{th}$ *quantum core* $QCore\_{c}$*in zero states along with ancilla to set as one****,*** *and store each* $P\_{k}\in P$*of size* $w=M\_{k}×log\_{2}\left|Σ\right|$*in register*$|P〉\_{DRk}$

$|ψ\_{n}〉\_{k}^{0}$ **:** $|T\_{0}〉\_{QAk}^{⊗n} |T\_{[0]}〉\_{QDk}^{⊗w} ⨂ |1〉\_{Qk}$ **&** $|P\_{\left[0 to M\_{k}-1\right]}〉\_{DRk}\leftarrow \left\{P\_{1},.,P\_{k},.,P\_{m}\right\}$ **(17)**

1. *Initialize superposition state in address register and ancilla of* $c^{th}$ *quantum core* $QCore\_{c}$ *as*

$|ψ\_{n}〉\_{k}^{1}$ **:** $H^{⊗(n+1)}\left(I^{⊗n} I^{⊗w} ⨂ I\right) |ψ\_{n}〉\_{k}^{0}$

 **:** $\frac{1}{\sqrt{N}}\sum\_{i=0}^{N- 1}|T\_{i}〉\_{QAk} ⨂ |T\_{[0]}〉\_{QDk} ⨂ \left(\frac{1}{\sqrt{2}}\left(|0〉-|1〉\right)\right)\_{Qk}⨂ |P〉\_{DRk}$

*Now, data and pattern register used at* $c^{th}$ *quantum core* $QCore\_{c}$ *is taking alongside for the further comparison, and separate the ancilla qubit, so we rewrite this state*$|ψ\_{n}〉\_{k}^{1}$ *as*

 **:** $\frac{1}{\sqrt{N}} \sum\_{i=0}^{N- 1}|T\_{i}〉\_{QAk} ⨂ |T\_{[0]}〉\_{QDk} ⨂ |P〉\_{DRk}⨂\left(\frac{1}{\sqrt{2}}\left(|0〉-|1〉\right)\right)\_{Qk}$

 **:** $\frac{1}{\sqrt{N}} \sum\_{i=0}^{N- 1}|T\_{i}〉\_{QAk} ⨂ |T\_{[0]}〉\_{QDk} ⨂ |P〉\_{DRk}⨂ |-〉\_{Qk}$ **(18)**

1. *For all* $|T\_{i}〉\_{QAk}$ *in their separate uniform quantum superposition state* $|ψ\_{n}〉\_{k}^{1}$***,*** *load data at* $|T\_{[i]}〉\_{QDk}$*as per entangled* $|T\_{i}〉\_{QAk}$ *by applying* $QMEM Transformation$ *of Eq. (1) as*

$$QMEM Transformation\leftarrow \left(U\_{QMEM}\leftarrow \left(U^{†}\_{Swap}\left(U\_{Load} \left(U\_{Swap}\right)\right)\right)\right)$$

*Unitary* $U\_{Load}$ *of Eq. (3) is directly applied for data transformation from memory to register*

$|ψ\_{n}〉\_{k}^{2}$ **:**$U\_{Load}\left(|ψ\_{n}〉\_{k}^{1}\right)$

**:**$ U\_{Load}\left(|T\_{i}〉\_{QAk} ⨂ |T\_{[0]}〉\_{QDk}\right)=U\_{Load}\left(|T\_{i}〉\_{QAk} ⨂ |T\_{\left[0\bigoplus\_{}^{}i\right]}〉\_{QDk}\right)$

 **:** $\frac{1}{\sqrt{N}} \sum\_{i=0}^{N- 1}|T\_{i}〉\_{QAk} ⨂ |T\_{[i]}〉\_{QDk} ⨂ |P〉\_{DRk}⨂ |-〉\_{Qk}$ **(19)**

*Repeat* $GSO$ *of Eq. (6) for*$Ο\left(\sqrt{{N}/{t\_{k} }}\right)$ *times in superposition*$|ψ\_{n}〉\_{k}^{2}$*by* $QEM Operation$*that is implicitly applied through*$U\_{kComp}$ *for exact matching of*$M\_{k}×log\_{2}\left|Σ\right|$*qubits size as*

$$GSO Opeartion\leftarrow \left(D\leftarrow U\_{Diff}\left(O\leftarrow \left(U\_{Mark}\left(U\_{Comp}\right)\right)\right)\right)$$

*Apply* $QEM Operation$ *by* $U\_{kComp}$ *of Eq. (5) to find index* $|T\_{i}〉\_{QAk}$*as to assure exact match*

$|ψ\_{n}〉\_{k}^{2}$ **:**$U\_{kComp}\left(|ψ\_{n}〉\_{k}^{2}\right)$

**:** $f\left(|T\_{i}〉\_{QAk}\right)=\left\{\begin{array}{c}\begin{matrix}0,&if \end{matrix}|T\_{[i to i+M\_{k}-1]}〉\_{QDk}\ne |P\_{\left[0 to M\_{k}-1\right]}〉\_{DRk}\\\begin{matrix}1,&if \end{matrix}|T\_{[i to i+M\_{k}-1]}〉\_{QDk}=|P\_{\left[0 to M\_{k}-1\right]}〉\_{DRk}\end{array}\right.$ **(20)**

$if\left(f\left(|T\_{i}〉\_{QAk}\right)= =1\right) then$

*Apply reflection* $U\_{Mark}$ *of Eq. (7) to mark* $|T\_{i}〉\_{QAk}$ *as* $\left(I-2|T\_{i}〉\_{QA}〈T\_{i}|\_{QA}\right) ⨂ \left(|T\_{i}〉\_{QA} ⨂ |q〉\right)$

$|ψ\_{n}〉\_{k}^{3}$**:** $U\_{Mark}\left(|ψ\_{n}〉\_{k}^{2}\right)$

 **:** $U\_{Mark}\left(\frac{1}{\sqrt{N}} \sum\_{i=0}^{N- 1}|T\_{i}〉\_{QAk} ⨂ |T\_{[i]}〉\_{QDk} ⨂ |P〉\_{DRk }⨂ |-〉\_{Qk}\right)$

 **:** $U\_{Mark}$*inverts*$|T\_{i}〉\_{QAk}$*to*$-|T\_{i}〉\_{QAk}$*if solution exist, otherwise*$\sum\_{\begin{array}{c}j \in N\\j \ne i \end{array}}^{}|T\_{j}〉\_{QAk}$*as*

 **:** $\left\{\begin{array}{c}\left(I-2|T\_{i}〉\_{QAk}〈T\_{i}|\_{QAk}\right)|T\_{i}〉\_{QAk}=|T\_{i}〉\_{QAk}-2|T\_{i}〉\_{QAk}\left⟨T\_{i}\right⟩\_{QAk}=-|T\_{i}〉\_{QAk}\\\left(I-2|T\_{i}〉\_{QAk}〈T\_{i}|\_{QAk}\right)|T\_{j}〉\_{QAk}=|T\_{i}〉\_{QAk}-2|T\_{i}〉\_{QAk}\left⟨T\_{j}\right⟩\_{QAk}= |T\_{i}〉\_{QAk}\end{array}\right.$

 **:** $\frac{1}{\sqrt{N}} \sum\_{i=0}^{N- 1}-|T\_{i}〉\_{QAk} ⨂ |T\_{[i]}〉\_{QDk} ⨂ |P〉\_{DRk }⨂ |-〉\_{Qk}$

*Therefore, each oracle call computes negated amplitude for index* $-|T\_{i}〉\_{QAk}$*to**obtain quantum state*$|ψ\_{n}〉\_{k}^{3}$ *from the passed state*$|ψ\_{n}〉\_{k}^{2}$ *to the oracle*

$O\left(\left|ψ\_{n}\right〉\_{k}^{3}\right)=\left(I-2\left|T\_{i}\right〉\_{QAk}\left〈T\_{i}\right|\_{QAk}\right) \left|ψ\_{n}\right〉\_{k}^{2}$

$=|ψ\_{n}〉\_{k}^{3}-2|T\_{i}〉\_{QAk} \left⟨ψ\_{n}\right⟩\_{QAk}$ ***Where,*** $\left⟨ψ\_{n}\right⟩\_{QAk}=\frac{1}{\sqrt{N}}$

$=|ψ\_{n}〉\_{k}^{3}-\frac{2}{\sqrt{N}}|T\_{i}〉\_{QAk}$**(21)**

*Apply diffusion operator*$ D\left(U\_{Diff}\left(O\right)\right)$*of Eq. (8) for the inversion about the mean as*

$D\left(U\_{Diff}\left(O\right)\right)=D\left(\left(2|ψ\_{n}〉〈ψ\_{n}|-I\right)\left(O\right)\right)=D\left(H^{⊗n}\left(2|0〉〈0|-I\right)H^{⊗n}\left(O\right)\right)$

$|ψ\_{n}〉\_{k}^{4}$**:** $U\_{Diff}\left(|ψ\_{n}〉\_{k}^{3}\right)$

 **:** $\left(2|ψ\_{n}〉\_{k}^{3}〈ψ\_{n}|\_{k}^{3}-I\right)\left(|ψ\_{n}〉\_{k}^{3}-\frac{2}{\sqrt{N}}|T\_{i}〉\_{QAk}\right)$

 **:** $2|ψ\_{n}〉\_{k}^{3}-|ψ\_{n}〉\_{k}^{3}-\frac{2^{2}}{N}|ψ\_{n}〉\_{k}^{3}+\frac{2}{\sqrt{N}} |T\_{i}〉\_{QAk}$

*Therefore, after each diffusion operation, the amplitude of marked index*$|T\_{i}〉\_{QAk}$*is increased by roughly*$\left(+\frac{2}{\sqrt{N}}\right)$*and finally on passing*$|ψ\_{n}〉\_{k}^{3}$*to diffusion, we get state* $|ψ\_{n}〉\_{k}^{4}$*as*

$|ψ\_{n}〉\_{k}^{4}$ **:**$D\left(|ψ\_{n}〉\_{k}^{3}\right)=|ψ\_{n}〉\_{k}^{3}-\frac{2^{2}}{N}|ψ\_{n}〉\_{k}^{3}+\frac{2}{\sqrt{N}} |T\_{i}〉\_{QAk}$**(22)**

***Trace Note – C2:*** *The operator*$GSO$*is repeated until required*$\left({π}/{4}\sqrt{{N}/{t\_{k} }}\right)$*number of iterations are not completed,**thus, this assures the final quantum state that must be containing a high probable solution at marked index* $|T\_{i}〉\_{QAk}$ *corresponding to*$ P\_{k}\in P$***.***

1. *Measure final state after the needed iterations as* $|ψ\_{n}〉\_{k}^{\left({π}/{4}\sqrt{{N}/{t\_{k} }}\right) }$*to get the desired index* $|T\_{i}〉\_{QAk}$*as high probable solution.*

***Trace Note – C3:*** *A quantum state gets collapsed after each measurement****,*** *so its repetition ensures to report all*$t\_{k}$*index location**of* $P\_{k}\in P$ *identified by* $c^{th}$ *quantum core* $QCore\_{c}$*as pattern match on behalf of output generated by* $k^{th}$ *exact search unitary* $U\_{kComp}$ *used in*$ GSO$***.***

1. *Verify for* $k^{th}$ *pattern using address register**of*$c^{th}$*core*$QCore\_{c}$*to report the pattern occurrence at index* $|T\_{i}〉\_{QAk}$*as*$|T\_{[i to i+M\_{k}-1]}〉\_{QDk}= =|P\_{k\left[0 to M\_{k}-1\right]}〉\_{DRk}$**(23)**