**APPENDIX D**

$Correctness Proof of Proposed Algorithm 2$**:** $EnQBCEA-MPM$

***Trace Note – D1:*** *Trace steps Eqs. (24) – (31) justifies* $Quantum Approximate Filtering$ *that identifies possible start / filtered location of each pattern*$ P\_{k}\in P$*using* $c^{th}$ *quantum core* $QCore\_{c}$ *accessing* $T$ *on shared*$ QMEM$***.*** *Iterate these number of steps until all likely* $t\_{k}$*indices corresponding to*$P\_{k}$ *be not stored in*$ LA\_{k}\left[…\right]$***.*** *Further, trace steps Eqs. (32) – (39) describes the**mathematical proof of correctness of* $EnQBCEA-MPM$*to ensure pattern match****.***

1. *Prepare each* $P\_{k}\in P$*of size* $w=M\_{k}×log\_{2}\left|Σ\right|$*in register*$|P〉\_{DRk}$ *of* $c^{th}$ *quantum core* $QCore\_{c}$*and then store* $|i\_{j}〉\_{k}$*in array* $SL\_{k}\left[M\right]$ *as preprocessed marking of possible start locations of distinct symbols for each pattern contained in register* $|P〉\_{DRk}$

$|P\_{\left[0 to M\_{k}-1\right]}〉\_{DRk}\leftarrow \left\{P\_{1},.,P\_{k},.,P\_{m}\right\}$ **&** $SL\_{k}\left[M\right]\leftarrow Distinct Symbols\left(P\_{k}\right)$ **(24)**

1. *Prepare quantum registers of each*$c^{th}$ *quantum core* $QCore\_{c}$*in zero states including address and auxiliary registers, and the pattern register is still kept initialized*

$|ψ\_{n}〉\_{k}^{0}$ **:** $|T\_{0}〉\_{QAk}^{⊗n} ⨂ |T\_{[0]}〉\_{QDk}^{⊗w} ⨂ |T\_{0}〉\_{AXk}^{⊗(n+1)}$$⨂$$|P\_{\left[0 to M\_{k}-1\right]}〉\_{DRk}$**(25)**

1. *Initialize superposition state in address register & entangle auxiliary register of* $QCore\_{c}$ *as*

$|ψ\_{n}〉\_{k}^{1}$ **:** $H^{⊗n}\left(I^{⊗n} I^{⊗w} ⨂ I^{⊗(n+1)}\right) |ψ\_{n}〉\_{k}^{0}$

 **:** $\frac{1}{\sqrt{N}} \sum\_{i=0}^{N- 1}|T\_{i}〉\_{QAk} ⨂ |T\_{[0]}〉\_{QDk} ⨂ |T\_{0}〉\_{AXk} ⨂ |P〉\_{DRk}$ **(26)**

1. *For all* $|T\_{i}〉\_{QAk}$ *in their separate uniform quantum superposition state* $|ψ\_{n}〉\_{k}^{1}$***,*** *load data at* $|T\_{[i]}〉\_{QDk}$*as per addresses*$|T\_{i}〉\_{QAk}$ *by applying* $QMEM Transformation$ *of Eq. (1) as*

$QMEM Transformation\leftarrow \left(U\_{QMEM}\leftarrow \left(U^{†}\_{Swap}\left(U\_{Load} \left(U\_{Swap}\right)\right)\right)\right)$

 *Unitary* $U\_{Load}$ *of Eq. (3) is directly applied for data transformation from memory to register*

$|ψ\_{n}〉\_{k}^{2}$**:**$U\_{Load}\left(|ψ\_{n}〉\_{k}^{1}\right)$

**:**$ U\_{Load}\left(|T\_{i}〉\_{QAk} ⨂ |T\_{[0]}〉\_{QDk}\right)=U\_{Load}\left(|T\_{i}〉\_{QAk} ⨂ |T\_{\left[0\bigoplus\_{}^{}i\right]}〉\_{QDk}\right)$

 **:** $\frac{1}{\sqrt{N}} \sum\_{i=0}^{N- 1}|T\_{i}〉\_{QAk} ⨂ |T\_{[i]}〉\_{QDk} ⨂ |T\_{0}〉\_{AXk}⨂ |P〉\_{DRk}$ **(27)**

1. *Apply unitary operator* $U\_{kSLoc}$*on* $|ψ\_{n}〉\_{k}^{2}$ *at*$QCore\_{c}$*to mark the occurrence of each distinct symbols of pattern as* $|i\_{j}〉\_{k}$ *in the entangled auxiliary register*$|T\_{i\_{j}}〉\_{AXk}$

$|ψ\_{n}〉\_{k}^{3}$**:**$U\_{kSLoc}\left(|ψ\_{n}〉\_{k}^{2}\right)$

 **:** $U\_{SLoc}\left(\frac{1}{\sqrt{N}} \sum\_{i=0}^{N- 1}|T\_{i}〉\_{QAk} ⨂ |T\_{[i]}〉\_{QDk} ⨂ |T\_{0}〉\_{AXk} ⨂ |P〉\_{DRk}\right)$

 **:** $\frac{1}{\sqrt{N}} \sum\_{i=0}^{N- 1}|T\_{i}〉\_{QAk} ⨂ |T\_{[i]}〉\_{QDk} ⨂ |T\_{i\_{j}}〉\_{AXk} ⨂ |P〉\_{DRk}$ **(28)**

1. *Apply unitary operator*$U\_{kPLoc}$*on*$|ψ\_{n}〉\_{k}^{3}$*at*$QCore\_{c}$*to get possible start location of* $|P〉\_{DRk}$*in auxiliary register*$|T\_{n+1}〉\_{AXk}$*with index position as*$|i-i\_{j}〉$

$|ψ\_{n}〉\_{k}^{4}$ **:**$U\_{kPLoc}\left(|ψ\_{n}〉\_{k}^{3}\right)$

 **:** $U\_{kPLoc}\left(\frac{1}{\sqrt{N}} \sum\_{i=0}^{N- 1}|T\_{i}〉\_{QAk} ⨂ |T\_{[i]}〉\_{QDk} ⨂ |T\_{i\_{j}}〉\_{AXk} ⨂ |P〉\_{DRk}\right)$

 **:** $\frac{1}{\sqrt{N}} \sum\_{i=0}^{N- 1}|T\_{i}〉\_{QAk} ⨂ |T\_{[i]}〉\_{QDk} ⨂ |T\_{i}-T\_{i\_{j}}〉\_{AXk} ⨂ |P〉\_{DRk}$ **(29)**

1. *Apply Hamming Distance at*$|T\_{i}-T\_{i\_{j}}〉\_{AXk}$ *to check for distance between text identified substring and pattern, such that,* $HD\leq $ ***threshold* (30)**
2. *Perform Hadamard operation at*$|T\_{i}〉\_{QAk}$*to zeroed its indices amplitudes and then merge amplitudes of entangled indices of*$|T\_{i}-T\_{i\_{j}}〉\_{AXk}$ *as*

$|ψ\_{n}〉\_{k}^{5}$**:**$H^{⊗n}\left(|ψ\_{n}〉\_{k}^{4}\right)$

**:** $H^{⊗n}\left(\frac{1}{\sqrt{N}} \sum\_{i=0}^{N- 1}α\_{i} |T\_{i}〉\_{QAk} ⨂ |T\_{[i]}〉\_{QDk} ⨂ α\_{i-i\_{j}} |T\_{i}-T\_{i\_{j}}〉\_{AXk} ⨂ |P〉\_{DRk}\right)$

 **:** $\frac{1}{\sqrt{N}} \sum\_{i=0}^{N- 1}α\_{i} |T\_{0}〉\_{QAk} ⨂ |T\_{[i]}〉\_{QDk} ⨂ α\_{i-i\_{j}} |T\_{i}-T\_{i\_{j}}〉\_{AXk} ⨂ |P〉\_{DRk}$ **(31)**

1. *Measure auxiliary*$|T\_{i}-T\_{i\_{j}}〉\_{AXk}$*and store the identified index as*$|T\_{i}〉$*in*$ LA\_{k}\left[…\right]$***.***

***Trace Note – D2:*** *As measurement destroys quantum state, so in each call at* $c^{th}$ *quantum core* $QCore\_{c}$ *on behalf of*$ P\_{k}\in P$*, the approximate filtering needs its execution several times to filter all*$t\_{k}$*indices location and then to store within the classical array*$ LA\_{k}\left[t\_{k}\right]$***.***

***Trace Note – D3:*** *Below mentioned tracing steps are repeated to find all*$t\_{k}$*exact occurrence of each*$P\_{k}\in P$*at* $c^{th}$ *quantum core* $QCore\_{c}$ *by using individual unitary operator* $U\_{kGetL}$ *that lets the address registers to access original filtered indices with the help of location register****.*** *So****,*** *further steps are written for*$EnQBCEA-MPM$*to report all occurrences of pattern*$ P\_{k}$***.***

1. *Prepare quantum registers of each*$c^{th}$ *quantum core* $QCore\_{c}$*in zero states along with ancilla to set as one****,*** *and store each* $P\_{k}\in P$*of size* $w=M\_{k}×log\_{2}\left|Σ\right|$*in register*$|P〉\_{DRk}$

$|ψ\_{tq}〉\_{k}^{0}$**:** $|T\_{0}〉\_{QLk}^{⊗tq} ⨂ |T\_{0}〉\_{QAk}^{⊗n} |T\_{[0]}〉\_{QDk}^{⊗w} ⨂ |1〉\_{Qk}$$⨂ |P〉\_{DRk}\leftarrow \left\{P\_{1},.,P\_{k},.,P\_{m}\right\}$ **(32)**

1. *Initialize superposition state in location register and ancilla of* $c^{th}$ *quantum core* $QCore\_{c}$ *as*

$|ψ\_{tq}〉\_{k}^{1}$ **:** $H^{⊗(tq+1)}\left(I^{⊗tq}⨂ I\right) |ψ\_{tq}〉\_{k}^{0}$

 **:** $\frac{1}{\sqrt{t\_{k}}}\sum\_{i=0}^{t\_{k}- 1}|T\_{i}〉\_{QLk} ⨂ |T\_{0}〉\_{QAk} ⨂ |T\_{[0]}〉\_{QDk} ⨂ \left(\frac{1}{\sqrt{2}}\left(|0〉-|1〉\right)\right)\_{Qk}⨂ |P〉\_{DRk}$

*Now, data and pattern register used at* $c^{th}$ *quantum core* $QCore\_{c}$ *is taking alongside for the further comparison, and separate the ancilla qubit, so we rewrite this state*$|ψ\_{tq}〉\_{k}^{1}$ *as*

 **:** $\frac{1}{\sqrt{t\_{k}}} \sum\_{i=0}^{t\_{k}- 1}|T\_{i}〉\_{QLk} ⨂ |T\_{0}〉\_{QAk} ⨂ |T\_{[0]}〉\_{QDk} ⨂ |P〉\_{DRk }⨂\left(\frac{1}{\sqrt{2}}\left(|0〉-|1〉\right)\right)\_{Qk}$

 **:** $\frac{1}{\sqrt{t\_{k}}} \sum\_{i=0}^{t\_{k}- 1}|T\_{i}〉\_{QLk} ⨂ |T\_{0}〉\_{QAk} ⨂ |T\_{[0]}〉\_{QDk} ⨂ |P〉\_{DRk }⨂ |-〉\_{Qk}$ **(33)**

1. *Apply an implicit unitary operator*$U\_{kGetL}$*to obtain the original filtered text index as*

$|ψ\_{tq}〉\_{k}^{2}$ **:** $U\_{kGetL}\left(|ψ\_{tq}〉\_{k}^{1}\right)$

$U\_{kGetL}$**:** $f\left(|T\_{iLk}〉\_{QLk}\right)=\left\{Gets |T\_{iLk}〉\_{QAk} as i^{th} Loc as \right.|T\_{|T\_{[i]}〉\_{QLk}}〉\_{QAk}\leftarrow |T\_{[i]}〉\_{QLk}=iLk$

 **:** $U\_{GetL}\left(\frac{1}{\sqrt{t\_{k}}} \sum\_{i=0}^{t\_{k}- 1}|T\_{i}〉\_{QLk} ⨂ |T\_{|T\_{[i]}〉\_{QLk}}〉\_{QAk} ⨂ |T\_{[0]}〉\_{QDk} ⨂ |P〉\_{DRk }⨂ |-〉\_{Qk}\right)$

 **:** $\frac{1}{\sqrt{t\_{k}}} \sum\_{i=0}^{t\_{k}- 1}|T\_{i}〉\_{QLk} ⨂ |T\_{iLk}〉\_{QAk} ⨂ |T\_{[0]}〉\_{QDk} ⨂ |P〉\_{DRk }⨂ |-〉\_{Qk}$ **(34)**

1. *For all* $|T\_{iLk}〉\_{QAk}$ *in their separate uniform quantum superposition state* $|ψ\_{tq}〉\_{k}^{2}$***,*** *load data at* $|T\_{[iLk]}〉\_{QDk}$*as per entangled*$|T\_{iLk}〉\_{QAk}$ *by applying* $QMEM Transformation$ *of Eq. (1)*

$$QMEM Transformation\leftarrow \left(U\_{QMEM}\leftarrow \left(U^{†}\_{Swap}\left(U\_{Load} \left(U\_{Swap}\right)\right)\right)\right)$$

 *Unitary* $U\_{Load}$ *of Eq. (3) is directly applied for data transformation from memory to register*

$|ψ\_{tq}〉\_{k}^{3}$**:**$U\_{Load}\left(|ψ\_{tq}〉\_{k}^{2}\right)$

**:**$ U\_{Load}\left(|T\_{iLk}〉\_{QAk} ⨂ |T\_{[0]}〉\_{QDk}\right)=U\_{Load}\left(|T\_{iLk}〉\_{QAk} ⨂ |T\_{\left[0\bigoplus\_{}^{}iLk\right]}〉\_{QDk}\right)$

 **:** $\frac{1}{\sqrt{t\_{k}}} \sum\_{iLk=0}^{t\_{k}- 1}|T\_{iLk}〉\_{QAk} ⨂ |T\_{[iLk]}〉\_{QDk} ⨂ |P〉\_{DRk }⨂ |-〉\_{Qk}$ **(35)**

*Repeat* $GSO$ *of Eq. (6) for*$Ο\left(\sqrt{{t\_{k}}/{t\_{k}^{'}}}\right)$ *times in superposition*$|ψ\_{tq}〉\_{k}^{3}$*by* $QEM Operation$*that is implicitly applied through*$U\_{kComp}$ *for exact matching of*$M\_{k}×log\_{2}\left|Σ\right|$*qubits size as*

$GSO Opeartion\leftarrow \left(D\leftarrow U\_{Diff}\left(O\leftarrow \left(U\_{Mark}\left(U\_{Comp}\right)\right)\right)\right)$

*Apply* $QEM Operation$ *by* $U\_{kComp}$ *of Eq. (5) to find index* $|T\_{iLk}〉\_{QAk}$*to assure exact match*

$|ψ\_{tq}〉\_{k}^{3}$ **:**$U\_{kComp}\left(|ψ\_{tq}〉\_{k}^{3}\right)$

**:** $f\left(|T\_{iLk}〉\_{QAk}\right)=\left\{\begin{array}{c}\begin{matrix}0,&if \end{matrix}|T\_{[iLk to iLk+M\_{k}-1]}〉\_{QDk}\ne |P\_{\left[0 to M\_{k}-1\right]}〉\_{DRk}\\\begin{matrix}1,&if \end{matrix}|T\_{[iLk to iLk+M\_{k}-1]}〉\_{QDk}=|P\_{\left[0 to M\_{k}-1\right]}〉\_{DRk}\end{array}\right.$ **(36)**

$if\left(f\left(|T\_{iLk}〉\_{QAk}\right)= =1\right) then$

*Apply unitary* $U\_{Mark}$ *of Eq. (7) to mark* $|T\_{iLk}〉\_{QAk}$ *as* $\left(I-2|T\_{i}〉\_{QA}〈T\_{i}|\_{QA}\right) ⨂ \left(|T\_{i}〉\_{QA} ⨂ |q〉\right)$

$|ψ\_{tq}〉\_{k}^{4}$ **:**$U\_{Mark}\left(|ψ\_{tq}〉\_{k}^{3}\right)$

 **:** $U\_{Mark}\left(\frac{1}{\sqrt{t\_{k}}} \sum\_{iLk=0}^{t\_{k}- 1}|T\_{iLk}〉\_{QAk} ⨂ |T\_{[iLk]}〉\_{QDk} ⨂ |P〉\_{DRk }⨂ |-〉\_{Qk}\right)$

 **:** $U\_{Mark}$*inverts*$|T\_{iLk}〉\_{QAk}$*to*$-|T\_{iLk}〉\_{QAk}$*if solution exist, otherwise*$\sum\_{\begin{array}{c}j \in t\_{k}\\j \ne iLk \end{array}}^{}|T\_{j}〉\_{QAk}$

 **:** $\left\{\begin{array}{c}\left(I-2|T\_{iLk}〉\_{QAk}〈T\_{iLk}|\_{QAk}\right)|T\_{iLk}〉\_{QAk}=|T\_{iLk}〉\_{QAk}-2|T\_{iLk}〉\_{QAk}\left⟨T\_{iLk}\right⟩\_{QAk}=-|T\_{iLk}〉\_{QAk}\\\left(I-2|T\_{iLk}〉\_{QAk}〈T\_{iLk}|\_{QAk}\right)|T\_{j}〉\_{QAk}=|T\_{iLk}〉\_{QAk}-2|T\_{iLk}〉\_{QAk}\left⟨T\_{j}\right⟩\_{QAk}= |T\_{iLk}〉\_{QAk}\end{array}\right.$

 **:** $\frac{1}{\sqrt{t\_{k}}} \sum\_{iL=0}^{t\_{k}- 1}-|T\_{iLk}〉\_{QAk} ⨂ |T\_{[iLk]}〉\_{QDk} ⨂ |P〉\_{DRk }⨂ |-〉\_{Qk}$

*Therefore, each oracle call computes negated amplitude for index* $-|T\_{iLk}〉\_{QAk}$*to**obtain quantum state*$|ψ\_{tq}〉\_{k}^{4}$ *from the passed state*$|ψ\_{tq}〉\_{k}^{3}$*to the oracle*

$O\left(|ψ\_{tq}〉\_{k}^{4}\right)=\left(I-2|T\_{iLk}〉\_{QAk}〈T\_{iLk}|\_{QAk}\right) |ψ\_{tq}〉\_{k}^{3}$

$=|ψ\_{tq}〉\_{k}^{3}-2|T\_{iLk}〉\_{QAk} \left⟨ψ\_{tq}\right⟩\_{QAk}$ ***Where,*** $\left⟨ψ\_{tq}\right⟩\_{QAk}=\frac{1}{\sqrt{t\_{k}}}$

$=|ψ\_{tq}〉\_{k}^{3}-\frac{2}{\sqrt{t\_{k}}}|T\_{iLk}〉\_{QAk}$**(37)**

*Apply diffusion operator*$ D\left(U\_{Diff}\left(O\right)\right)$*of Eq. (8) for the inversion about the mean as*

$D\left(U\_{Diff}\left(O\right)\right)=D\left(\left(2|ψ\_{n}〉〈ψ\_{n}|-I\right)\left(O\right)\right)=D\left(H^{⊗n}\left(2|0〉〈0|-I\right)H^{⊗n}\left(O\right)\right)$

$|ψ\_{tq}〉\_{k}^{5}$**:** $U\_{Diff}\left(|ψ\_{tq}〉\_{k}^{4}\right)$

 **:** $\left(2|ψ\_{tq}〉\_{k}^{4}〈ψ\_{tq}|\_{k}^{4}-I\right)\left(|ψ\_{tq}〉\_{k}^{4}-\frac{2}{\sqrt{t\_{k}}}|T\_{iLk}〉\_{QAk}\right)$

 **:** $2|ψ\_{tq}〉\_{k}^{4}-|ψ\_{tq}〉\_{k}^{4}-\frac{2^{2}}{t\_{k}}|ψ\_{tq}〉\_{k}^{4}+\frac{2}{\sqrt{t\_{k}}} |T\_{iLk}〉\_{QAk}$

*Therefore, after each diffusion operation, the amplitude of marked index*$|T\_{iLk}〉\_{QAk}$*increases by roughly*$\left(+\frac{2}{\sqrt{t\_{k}}}\right)$*and finally on passing*$|ψ\_{tq}〉\_{k}^{4}$*to diffusion, we get state* $|ψ\_{tq}〉\_{k}^{5}$*as*

$|ψ\_{tq}〉\_{k}^{5}$ **:**$D\left(|ψ\_{tq}〉\_{k}^{4}\right)=|ψ\_{tq}〉\_{k}^{4}-\frac{2^{2}}{t\_{k}}|ψ\_{tq}〉\_{k}^{4}+\frac{2}{\sqrt{t\_{k}}} |T\_{iLk}〉\_{QAk}$**(38)**

***Trace Note – D4:*** *The operator*$GSO$*is repeated until required*$\left({π}/{4}\sqrt{{t\_{k}}/{t\_{k}^{'}}}\right)$*number of iterations are not completed;**thus, this assures the final quantum state that must be containing a high probable solution at marked index*$ |T\_{iLk}〉\_{QAk}$ *corresponding to*$ P\_{k}\in P$***.***

1. *Measure final state after the needed iterations as* $|ψ\_{tq}〉\_{k}^{\left({π}/{4}\sqrt{{t\_{k}}/{t\_{k}^{'}}}\right) }$*to get the desired index* $|T\_{iLk}〉\_{QAk}$*as high probable solution.*

***Trace Note – D5:*** *The quantum state gets collapsed after each measurement****,*** *so its repetition will ensure to report all*$t\_{k}^{'}$ *index location**of* $P\_{k}\in P$ *identified by the* $c^{th}$ *quantum core* $QCore\_{c}$*as pattern match over likely* $t\_{k}$ *filtered indices stored in*$ LA\_{k}\left[t\_{k}\right]$***.*** *Still, the output based on behalf of* $k^{th}$ *exact search unitary* $U\_{kComp}$ *is used within the*$ GSO$ *operator****.***

1. *Verify for* $k^{th}$ *pattern using address register**of*$c^{th}$*core*$QCore\_{c}$*to report the pattern occurrence at index*$|T\_{iLk}〉\_{QAk}$*as* $|T\_{[iLk to iLk+M\_{k}-1]}〉\_{QDk}= =|P\_{\left[0 to M\_{k}-1\right]}〉\_{DRk}$**(39)**