

Modelling and analysis of complement system signalling pathways: Roles of C3, C5a and pro-inflammatory cytokines in SARS-CoV-2 infection

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The Computational Tree Logic (CTL) formalism fundamentals (Paracha, 2015; Chabrier and Fages, 2003)

The CTL formula is defined inductively with:

- A finite state of propositional variables: $\{v_i\}$.
- Logical connectives: \top (boolean true), \perp (boolean false), \neg (negation), \wedge (logical AND), \vee (logical OR), \Rightarrow (implication).
- Temporal connectives: AX, EX, AG, EG, AU, EU, AF, EF.
- Rules of formation: v_i, \top, \perp are formulae. If ϕ_1 and ϕ_2 are propositional logic formulae then $(\neg\phi_1), (\phi_1 \wedge \phi_2), (\phi_1 \vee \phi_2), (\phi_1 \Rightarrow \phi_2), AX\phi_1, EX\phi_1, A[\phi_1 U \phi_2], E[\phi_1 U \phi_2], AG\phi_1, EG\phi_1, AF\phi_1, EF\phi_1$ are formulae.

The logical connectives are the classical ones: false, true, not, and, or implication. All the temporal connectives are pairs of symbols. The first element of the pair is A or E. The second one is temporal operator as X, F, G or U.

Meaning of the Connectives:

- A : along All paths
- E : along at least one path (there Exist)
- X : neXt state
- F (also written as ' \diamond '): some Future state
- G (also written as ' \square ') : all future states (Globally)
- U : Until

All the CTL connectives are equivalent to a combination of a set of six of them, for example $\{\perp, \neg, \wedge, AF, EU, EX\}$. So, all the formulas can be written with this set of connectives using the following equivalences.

Equivalences between CTL Formulae

1. $\neg AF\phi \equiv EG\neg\phi$
2. $\neg EF\phi \equiv AG\neg\phi$

3. $\neg AX\phi \equiv EX\neg\phi$
4. $AF\phi \equiv A[\top \cup \phi]$
5. $EF\phi \equiv E[\top \cup \phi]$
6. $A[p \cup q] \equiv \neg(E[\neg q U (\neg p \wedge \neg q)] \vee EG\neg q)$

Establishment of CTL:

- State formulae syntax as $\phi := \text{true } v_i, \phi_1 \wedge \phi_2, \neg\phi_1, E\alpha, A\alpha$, where: ϕ_1, ϕ_2 are state formulae, α : is path formula.
- Path formulae syntax as $\alpha := \phi, X\alpha_1, \alpha_1 U \alpha_2, F\alpha_1, G\alpha_1$, where: ϕ is state formula, $\alpha_1 \alpha_2$: are path formulae.
- We can write path formulae as state formula for development of CTL as $X\phi_1, \phi_1 U \phi_2, F\phi_1, G\phi_1$.
- So, with the help of above governing rules we can write CTLs, some examples of legal and illegal CTLs are given as
 - Legal CTL formulae: $EFv_1, EFAGv_1, AXv_2, AFv_1 \wedge AGv_2, A(v_1 U (FGv_2))$.
 - Illegal CTL formulae: $AFGv_1, Av_1, EGFv_i, A(Fv_1 \wedge Gv_2), A(v_1 U (Gv_2))$.

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