

Supplemental File for “Efficiently Handling Constraints in Mixed-Integer Nonlinear Programming Problems Using Gradient-Based Repair Differential Evolution”

S-I. TEST PROBLEMS

F1 : minimize: $f(x,y) = (x-1)^2 + (y-3)^2$

subject to:

$$g(x,y) = (x+1)^2 + (y+1)^2 - 1 \leq 0$$

$$x \in [-3, 1]$$

$$y \in \{-3, -2, -1, 0, 1\}$$

The optimal solution is $x^* = -1$ and $y^* = 0$, and $f(x^*, y^*) = 13.0000$.

F2 : minimize: $f(x,y) = x^2 + (y_1 - 1)^2 + (y_2 - 2)^2$

subject to:

$$g(x,y) = x^2 + y_1^2 + 0.5y_2^2 - 1.5 \leq 0$$

$$x \in [-1, 100]$$

$$y_1, y_2 \in \{-1, 0, \dots, 100\}$$

The optimal solution is $x^* = 0$ and $y^* = (1, 1)$, and $f(x^*, y^*) = 1.0000$.

F3 : minimize: $f(x,y) = -x - y$

subject to:

$$g_1(x,y) = -x + y - 2.005 \leq 0$$

$$g_2(x,y) = x - y + 0.5 \leq 0$$

$$g_3(x,y) = 0.505x + y - 3.505 \leq 0$$

$$x \in [-1, 100]$$

$$y \in \{-1, 0, \dots, 100\}$$

The optimal solution is $x^* = 1$ and $y^* = 3$, and $f(x^*, y^*) = -4.0000$.

F4 : minimize: $f(x,y) = -x - y$

subject to:

$$g_1(x,y) = y - 3.4 \leq 0$$

$$g_2(x,y) = x - y \leq 0$$

$$x \in [-1, 100]$$

$$y \in \{-1, 0, \dots, 100\}$$

The optimal solution is $x^* = 3$ and $y^* = 3$, and $f(x^*, y^*) = -6.0000$.

F5 : minimize: $f(x,y) = (x-0.5)^2 + (y-1)^2$

subject to:

$$h(x,y) = -x^2 + y = 0$$

$$x \in [-1, 3.1]$$

$$y \in \{-1, 0, \dots, 4\}$$

The best known optimal solution is $x^* = 1$ and $y^* = 1$, and $f(x^*, y^*) = 0.2500$.

F6 : minimize: $f(x,y) = (x-10)^3 + (y-20)^3$

subject to:

$$g_1(x,y) = -(x-5)^2 - (y-4.86)^2 + 100 \leq 0$$

$$g_2(x,y) = (x-8)^2 + (y-5.48)^2 - 60 \leq 0$$

$$x \in [-1, 100]$$

$$y \in \{-1, 0, \dots, 100\}$$

The best known optimal solution is $x^* = 14.22498780$ and $y^* = 1$, and $f(x^*, y^*) = -6783.5818$.

F7 : minimize: $f(\mathbf{x}, \mathbf{y}) = \exp(x_1 x_2 x_3 y_1 y_2)$

subject to:

$$h_1(\mathbf{x}, \mathbf{y}) = x_1^2 + x_2^2 + x_3^2 + y_1^2 + y_2^2 - 10 = 0$$

$$h_2(\mathbf{x}, \mathbf{y}) = x_2 y_1 - 5 x_3 y_2 = 0$$

$$h_3(\mathbf{x}, \mathbf{y}) = x_1^3 + y_1^3 + 1 = 0$$

$$x_1 \in [-2.3, 2.3]$$

$$x_2, x_3 \in [-3.2, 3.2]$$

$$y_1 \in \{-2, -1, 0, 1, 2\}$$

$$y_2 \in \{-3, -2, -1, 0, 1, 2, 3\}$$

The best known optimal solution is $\mathbf{x}^* = (-1.25994205, -2.48314049, 0.496648098)$ and $\mathbf{y}^* = (1, -1)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 0.2114$.

F8 : minimize: $f(\mathbf{x}, \mathbf{y}) = x_1 + x_2 + y_1$

subject to:

$$g_1(\mathbf{x}, \mathbf{y}) = -1 + 0.0025(x_3 + x_4) \leq 0$$

$$g_2(\mathbf{x}, \mathbf{y}) = -1 + 0.0025(-x_3 + y_2 + y_3) \leq 0$$

$$g_3(\mathbf{x}, \mathbf{y}) = -1 + 0.01(x_5 - y_2) \leq 0$$

$$g_4(\mathbf{x}, \mathbf{y}) = -x_1 x_4 + 833.33252 x_3 + 100 x_1 - 83333.333 \leq 0$$

$$g_5(\mathbf{x}, \mathbf{y}) = -y_1 y_3 + 1250 y_2 + x_3 y_1 - 1250 x_3 \leq 0$$

$$g_6(\mathbf{x}, \mathbf{y}) = -x_2 x_5 + 1250000 + x_2 y_2 - 2500 y_2 \leq 0$$

$$x_1 \in [100, 10000]$$

$$x_2 \in [1000, 10000]$$

$$x_3, x_4, x_5 \in [10, 1000]$$

$$y_1 \in \{1000, 1020, \dots, 10000\}$$

$$y_2, y_3 \in \{20, 40, \dots, 1000\}$$

The best known optimal solution is $\mathbf{x}^* = (555.55433833, 5000, 180, 220, 400)$ and $\mathbf{y}^* = (1500, 300, 280)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 7055.5544$.

F9 : minimize: $f(\mathbf{x}, \mathbf{y}) = x_1 + x_2 + y_1$

subject to:

$$\begin{aligned} g_1(\mathbf{x}, \mathbf{y}) &= -1 + 0.0025(x_3 + x_4) \leq 0 \\ g_2(\mathbf{x}, \mathbf{y}) &= -1 + 0.0025(-x_3 + y_2 + y_3) \leq 0 \\ g_3(\mathbf{x}, \mathbf{y}) &= -1 + 0.01(x_5 - y_2) \leq 0 \\ g_4(\mathbf{x}, \mathbf{y}) &= -x_1x_4 + 833.33252x_3 + 100x_1 - 83333.333 \leq 0 \\ g_5(\mathbf{x}, \mathbf{y}) &= -y_1y_3 + 1250y_2 + x_3y_1 - 1250x_3 \leq 0 \\ g_6(\mathbf{x}, \mathbf{y}) &= -x_2x_5 + 1250000 + x_2y_2 - 2500y_2 \leq 0 \\ x_1 &\in [100, 10000], x_2 \in [1000, 10000] \\ x_3, x_4, x_5 &\in [10, 1000] \\ y_1 &\in \{1000, 1050, \dots, 10000\} \\ y_2, y_3 &\in \{50, 100, \dots, 1000\} \end{aligned}$$

The best known optimal solution is $\mathbf{x}^* = (833.33171, 5000, 200, 200, 400)$ and $\mathbf{y}^* = (1250, 300, 300)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 7083.3317$.

F10 : minimize: $f(\mathbf{x}, \mathbf{y}) = x_1 + x_2 + y_1$

subject to:

$$\begin{aligned} g_1(\mathbf{x}, \mathbf{y}) &= -1 + 0.0025(x_3 + x_4) \leq 0 \\ g_2(\mathbf{x}, \mathbf{y}) &= -1 + 0.0025(-x_3 + y_2 + y_3) \leq 0 \\ g_3(\mathbf{x}, \mathbf{y}) &= -1 + 0.01(x_5 - y_2) \leq 0 \\ g_4(\mathbf{x}, \mathbf{y}) &= -x_1x_4 + 833.33252x_3 + 100x_1 - 83333.333 \leq 0 \\ g_5(\mathbf{x}, \mathbf{y}) &= -y_1y_3 + 1250y_2 + x_3y_1 - 1250x_3 \leq 0 \\ g_6(\mathbf{x}, \mathbf{y}) &= -x_2x_5 + 1250000 + x_2y_2 - 2500y_2 \leq 0 \\ x_1 &\in [100, 10000], x_2 \in [1000, 10000] \\ x_3, x_4, x_5 &\in [10, 1000] \\ y_1 &\in \{1000, 1100, \dots, 10000\} \\ y_2, y_3 &\in \{100, 200, \dots, 1000\} \end{aligned}$$

The best known optimal solution is $\mathbf{x}^* = (833.33171, 5000, 200, 200, 400)$ and $\mathbf{y}^* = (1300, 300, 300)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 7133.3317$.

F11 : minimize: $f(\mathbf{x}) = \sum_{j=1}^5 \sum_{i=1}^5 c_{ij}x_{(10+i)}x_{(10+j)} + 2 \sum_{j=1}^5 d_jx_{(10+j)}^3 - \sum_{i=1}^{10} b_ix_i$

subject to:

$$\begin{aligned} g_j(\mathbf{x}) &= -2 \sum_{i=1}^5 c_{ij}x_{(10+i)} - 3d_jx_{(10+j)}^2 - e_j + \sum_{i=1}^{10} a_{ij}x_i \leq 0, \\ j &= 1, \dots, 5 \\ x_1, x_2, x_4, x_6, \dots, x_{11}, x_{13}, x_{14}, x_{15} &\in [0, 10] \\ x_3, x_5, x_{12} &\in \{0, 1, \dots, 10\} \end{aligned}$$

where $\mathbf{b} = [-40, -2, -0.25, -4, -4, -1, -40, -60, 5, 1]$. The other data can be found in Table S-I. The best known optimal solution is $\mathbf{x}^* = (x_1, x_2, x_4, x_6, \dots, x_{11}, x_{13}, x_{14}, x_{15}) = (0, 0, 0, 9.99999985, 0, 0,$

0, 0, 0.28879805, 0.43951302, 0.31935496, 0.44885950) and $\mathbf{y}^* = (x_3, x_5, x_{12}) = (4, 4, 0)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 33.5066$.

$$\mathbf{F12} : \text{minimize: } f(\mathbf{x}) = \sum_{j=1}^5 \sum_{i=1}^5 c_{ij} x_{(10+i)} x_{(10+j)} + 2 \sum_{j=1}^5 d_j x_{(10+j)}^3 - \sum_{i=1}^{10} b_i x_i$$

subject to:

$$g_j(\mathbf{x}) = -2 \sum_{i=1}^5 c_{ij} x_{(10+i)} - 3d_j x_{(10+j)}^2 - e_j + \sum_{i=1}^{10} a_{ij} x_i \leq 0,$$

$$j = 1, \dots, 5$$

$$x_1, x_2, x_4, x_6, \dots, x_9, x_{11}, x_{13}, x_{14} \in [0, 10]$$

$$x_3, x_5, x_{10}, x_{12}, x_{15} \in \{0, 1, \dots, 10\}$$

where $\mathbf{b} = [-40, -2, -0.25, -4, -4, -1, -40, -60, 5, 1]$. The other data can be found in Table S-I. The best known optimal solution is $\mathbf{x}^* = (x_1, x_2, x_4, x_6, \dots, x_9, x_{11}, x_{13}, x_{14}) = (0, 0, 0, 9.99999999, 0, 0, 2.96750117, 0.39963905, 0.82151768, 0.64848398)$, and $\mathbf{y}^* = (x_3, x_5, x_{10}, x_{12}, x_{15}) = (2, 4, 0, 0, 1)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 41.7399$.

$$\mathbf{F13} : \text{minimize: } f(\mathbf{x}, \mathbf{y}) = f_1(y_1) + f_2(x_1)$$

where:

$$f_1(y_1) = \begin{cases} 30y_1, & 0 \leq y_1 \leq 300 \\ 31y_1, & 300 \leq y_1 \leq 400 \end{cases}$$

$$f_2(x_1) = \begin{cases} 28x_1, & 0 \leq x_1 \leq 100 \\ 29x_1, & 100 \leq x_1 \leq 200 \\ 30x_1, & 200 \leq x_1 \leq 1000 \end{cases}$$

subject to:

$$h_1(\mathbf{x}, \mathbf{y}) = -y_1 + 300 - \frac{y_2 x_2}{131.078} \cos(1.48477 - x_4) + \frac{0.90798 y_2^2}{131.078} \cos(1.47588) = 0$$

$$h_2(\mathbf{x}, \mathbf{y}) = -x_1 - \frac{y_2 x_2}{131.078} \cos(1.48477 + x_4) + \frac{0.90798 x_2^2}{\cos} \frac{131.078}{(1.47588)} = 0$$

$$h_3(\mathbf{x}, \mathbf{y}) = -x_3 - \frac{y_2 x_2}{131.078} \sin(1.48477 + x_4) + \frac{0.90798 x_2^2}{131.078} \sin(1.47588) = 0$$

$$h_4(\mathbf{x}, \mathbf{y}) = 200 - \frac{y_2 x_2}{131.078} \sin(1.48477 - x_4) + \frac{0.90798 y_2^2}{131.078} \sin(1.47588) = 0$$

$$x_1 \in [0, 1000], x_2 \in [340, 420]$$

$$x_3 \in [-1000, 1000], x_4 \in [0, 0.5236]$$

$$y_1 \in \{0, 20, \dots, 400\}, y_2 \in \{340, 360, \dots, 420\}$$

The best known optimal solution is $\mathbf{x}^* = (81.57454322, 416.85149297, -9.77394390, 0.05912763)$ and $\mathbf{y}^* = (220, 380)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 8884.0872$.

Table S-I. Data for problems F11 and F12

j	1	2	3	4	5
e_j	-15	-27	-36	-18	-12
c_{1j}	30	-20	-10	32	-10
c_{2j}	-20	39	-6	-31	32
c_{3j}	-10	-6	10	-6	-10
c_{4j}	32	-31	-6	39	-20
c_{5j}	-10	32	-10	-20	30
d_j	4	8	10	6	2
a_{1j}	-16	2	0	1	0
a_{2j}	0	-2	0	0.4	2
a_{3j}	-3.5	0	2	0	0
a_{4j}	0	-2	0	-4	-1
a_{5j}	0	-9	-2	1	-2.8
a_{6j}	2	0	-4	0	0
a_{7j}	-1	-1	-1	-1	-1
a_{8j}	-1	-2	-3	-2	-1
a_{9j}	1	2	3	4	5
a_{10j}	1	1	1	1	1

F14 : minimize: $f(\mathbf{x}, \mathbf{y}) = f_1(y_1) + f_2(x_1)$

where:

$$f_1(y_1) = \begin{cases} 30y_1, & 0 \leq y_1 \leq 300 \\ 31y_1, & 300 \leq y_1 \leq 400 \end{cases}$$

$$f_2(x_1) = \begin{cases} 28x_1, & 0 \leq x_1 \leq 100 \\ 29x_1, & 100 \leq x_1 \leq 200 \\ 30x_1, & 200 \leq x_1 \leq 1000 \end{cases}$$

subject to:

$$h_1(\mathbf{x}, \mathbf{y}) = -y_1 + 300 - \frac{y_2 x_2}{131.078} \cos(1.48477 - x_4) + \frac{0.90798 y_2^2}{131.078} \cos(1.47588) = 0$$

$$h_2(\mathbf{x}, \mathbf{y}) = -x_1 - \frac{y_2 x_2}{131.078} \cos(1.48477 + x_4) + \frac{0.90798 x_2^2}{131.078} \cos(1.47588) = 0$$

$$h_3(\mathbf{x}, \mathbf{y}) = -x_3 - \frac{y_2 x_2}{131.078} \sin(1.48477 + x_4) + \frac{0.90798 x_2^2}{131.078} \sin(1.47588) = 0$$

$$h_4(\mathbf{x}, \mathbf{y}) = 200 - \frac{y_2 x_2}{131.078} \sin(1.48477 - x_4) \\ + \frac{0.90798 y_2^2}{131.078} \sin(1.47588) = 0$$

$$x_1 \in [0, 1000], x_2 \in [340, 420]$$

$$x_3 \in [-1000, 1000], x_4 \in [0, 0.5236]$$

$$y_1 \in \{0, 50, \dots, 400\}, y_2 \in \{350, 400\}$$

The best known optimal solution is $\mathbf{x}^* = (51.69905661, 394.30118556, 20.47601024, 0.03816719)$ and $\mathbf{y}^* = (250, 350)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 8947.5736$.

$$\mathbf{F15} : \text{minimize} : f(\mathbf{x}, \mathbf{y}) = x_1^2 + y_1^2 + x_1 y_1 - 14x_1 - 16y_1 + (y_2 - 10)^2 + 4(x_2 - 5)^2 + (x_3 - 3)^2 \\ + 2(x_4 - 1)^2 + 5x_5^2 + 7(x_6 - 11)^2 + 2(y_3 - 10)^2 + (x_7 - 7)^2 + 45$$

subject to:

$$g_1(\mathbf{x}, \mathbf{y}) = -105 + 4x_1 + 5y_1 - 3x_5 + 9x_6 \leq 0$$

$$g_2(\mathbf{x}, \mathbf{y}) = 10x_1 - 8y_1 - 17x_5 + 2x_6 \leq 0$$

$$g_3(\mathbf{x}, \mathbf{y}) = -8x_1 + 2y_1 + 5y_3 - 2x_7 - 12 \leq 0$$

$$g_4(\mathbf{x}, \mathbf{y}) = 3(x_1 - 2)^2 + 4(y_1 - 3)^2 + 2y_2^2 - 7x_2 - 120 \leq 0$$

$$g_5(\mathbf{x}, \mathbf{y}) = 5x_1^2 + 8y_1 + (y_2 - 6)^2 - 2x_2 - 40 \leq 0$$

$$g_6(\mathbf{x}, \mathbf{y}) = x_1^2 + 2(y_1 - 2)^2 - 2x_1 y_1 + 14x_3 - 6x_4 \leq 0$$

$$g_7(\mathbf{x}, \mathbf{y}) = 0.5(x_1 - 8)^2 + 2(y_1 - 4)^2 + 3x_3^2 - x_4^2 - 30 \leq 0$$

$$g_8(\mathbf{x}, \mathbf{y}) = -3x_1 + 6y_1 + 12(y_3 - 8)^2 - 7x_7 \leq 0$$

$$x_1, x_2, \dots, x_7 \in [-10, 10]$$

$$y_1, y_2, y_3 \in \{-10, -9, \dots, 10\}$$

The best known optimal solution is $\mathbf{x}^* = (2.45799944, 5.10440319, 0.89287364, 1.45166575, 1.68117614, 9.99999999, 8.66800226)$ and $\mathbf{y}^* = (2, 8, 9)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 28.3514$.

$$\mathbf{F16} : \text{minimize} : f(\mathbf{x}, \mathbf{y}) = x_1^2 + y_1^2 + x_1 y_1 - 14x_1 - 16y_1 + (y_2 - 10)^2 + 4(x_2 - 5)^2 + (y_3 - 3)^2 \\ + 2(x_3 - 1)^2 + 5x_4^2 + 7(y_4 - 11)^2 + 2(y_5 - 10)^2 + (x_5 - 7)^2 + 45$$

subject to:

$$g_1(\mathbf{x}, \mathbf{y}) = -105 + 4x_1 + 5y_1 - 3x_4 + 9x_5 \leq 0$$

$$g_2(\mathbf{x}, \mathbf{y}) = 10x_1 - 8y_1 - 17x_4 + 2y_4 \leq 0$$

$$g_3(\mathbf{x}, \mathbf{y}) = -8x_1 + 2y_1 + 5y_5 - 2x_5 - 12 \leq 0$$

$$g_4(\mathbf{x}, \mathbf{y}) = 3(x_1 - 2)^2 + 4(y_1 - 3)^2 + 2y_2^2 - 7x_2 - 120 \leq 0$$

$$g_5(\mathbf{x}, \mathbf{y}) = 5x_1^2 + 8y_1 + (y_2 - 6)^2 - 2x_2 - 40 \leq 0$$

$$g_6(\mathbf{x}, \mathbf{y}) = x_1^2 + 2(y_1 - 2)^2 - 2x_1 y_1 + 14y_3 - 6x_3 \leq 0$$

$$g_7(\mathbf{x}, \mathbf{y}) = 0.5(x_1 - 8)^2 + 2(y_1 - 4)^2 + 3y_3^2 - x_3^2 - 30 \leq 0$$

$$g_8(\mathbf{x}, \mathbf{y}) = -3x_1 + 6y_1 + 12(y_5 - 8)^2 - 7x_5 \leq 0$$

$$x_1, x_2, \dots, x_5 \in [-10, 10]$$

$$y_1, y_2, \dots, y_5 \in \{-10, -9, \dots, 10\}$$

The best known optimal solution is $\mathbf{x}^* = (2.45787583, 5.10288399, 1.70160838, 1.68110343, 8.66849668)$ and $\mathbf{y}^* = (2, 8, 1, 10, 9)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 28.4879$.

$$\mathbf{F17} : \text{minimize} : f(\mathbf{x}, \mathbf{y}) = 10x_1 - 7x_3 - 18\log(x_2 + 1) - 19.2\log(x_1 - x_2 + 1) + 5y_4 + 6y_5 + 8y_6 + 10$$

subject to:

$$\begin{aligned} g_1(\mathbf{x}, \mathbf{y}) &= -0.8\log(x_2 + 1) - 0.96\log(x_1 - x_2 + 1) + 0.8x_3 \leq 0 \\ g_2(\mathbf{x}, \mathbf{y}) &= x_2 - x_1 \leq 0 \\ g_3(\mathbf{x}, \mathbf{y}) &= x_2 - 2y_4 \leq 0 \\ g_4(\mathbf{x}, \mathbf{y}) &= x_1 - x_2 - 2y_5 \leq 0 \\ g_5(\mathbf{x}, \mathbf{y}) &= -\log(x_2 + 1) - 1.2\log(x_1 - x_2 + 1) + x_3 + 2y_6 - 2 \leq 0 \\ g_6(\mathbf{x}, \mathbf{y}) &= y_4 + y_5 - 1 \leq 0 \\ x_1, x_2 &\in [0, 2], x_3 \in [0, 1] \\ y_4, y_5, y_6 &\in \{0, 1\} \end{aligned}$$

The best known optimal solution is $\mathbf{x}^* = (1.3010, 0, 1)$ and $\mathbf{y}^* = (0, 1, 0)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 6.0098$.

$$\begin{aligned} \mathbf{F18} : \text{minimize} : f(\mathbf{x}, \mathbf{y}) &= -0.29x_1 - 0.43x_2 - 0.015x_3 - 0.985x_4 - 0.165x_5 - 0.105x_6 \\ &\quad - 0.37x_7 - 0.2x_8 - 0.49x_9 - 0.34x_{10} - 0.175y_{11} - 0.39y_{12} - 0.83y_{13} \\ &\quad - 0.805y_{14} - 0.06y_{15} - 0.4y_{16} - 0.52y_{17} - 0.415y_{18} - 0.655y_{19} - 0.63y_{20} \end{aligned}$$

subject to:

$$\begin{aligned} g_1(\mathbf{x}, \mathbf{y}) &= \sqrt{0.0001 + \sum_{i=1}^{10} x_i^2 + \sum_{j=11}^{20} y_j^2} - 10 \leq 0 \\ x_1, x_2, \dots, x_{10} &\in [0, 5], y_{11}, y_{12}, \dots, y_{20} \in \{0, 1, \dots, 5\} \end{aligned}$$

The best known optimal solution is $\mathbf{x}^* = (1.2381, 1.8357, 0.0640, 4.2057, 0.7045, 0.4482, 1.5800, 0.8537, 2.0920, 1.4515)$ and $\mathbf{y}^* = (1, 2, 4, 4, 0, 2, 2, 2, 3, 3)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = -21.7491$.

$$\mathbf{F19} : \text{minimize} : f(\mathbf{x}, \mathbf{y}) = 7.5y_3 + 5.5y_4 + 7x_5 + 6x_6 + 5(x_1 + x_2)$$

subject to:

$$\begin{aligned} h_1(\mathbf{x}, \mathbf{y}) &= y_3 + y_4 - 1 = 0 \\ h_2(\mathbf{x}, \mathbf{y}) &= 0.9(1 - e^{-0.5x_5})x_1 + 0.8(1 - e^{-0.4x_6})x_2 = 10 \\ h_3(\mathbf{x}, \mathbf{y}) &= 0.9(1 - e^{-0.5x_5})x_1y_3 + 0.8(1 - e^{-0.4x_6})x_2y_4 = 10 \\ g_1(\mathbf{x}, \mathbf{y}) &= x_1 - 20x_3 \leq 0 \\ g_2(\mathbf{x}, \mathbf{y}) &= x_2 - 20x_4 \leq 0 \\ g_3(\mathbf{x}, \mathbf{y}) &= x_5 - 10x_3 \leq 0 \\ g_4(\mathbf{x}, \mathbf{y}) &= x_6 - 10x_4 \leq 0 \\ g_5(\mathbf{x}, \mathbf{y}) &= -0.9(1 - e^{-0.5x_5})x_1 \leq 0 \\ g_6(\mathbf{x}, \mathbf{y}) &= -0.8(1 - e^{-0.4x_6})x_2 \leq 0 \\ x_1, x_5, x_6 &\in [0, 100], x_2 \in [0, 50] \\ y_3, y_4 &\in \{0, 1\} \end{aligned}$$

The best known optimal solution is $\mathbf{x}^* = (13.4279, 0, 3.5142, 0)$ and $\mathbf{y}^* = (1, 0)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 99.2390$.

F20 : minimize : $f(x, y) = -0.00201x_1^2y_2^4y_3$
subject to:

$$\begin{aligned} g_1(x, y) &= y_2^2y_3 - 675 \leq 0 \\ g_2(x, y) &= 0.1x_1^2y_2^2 - 0.419 \leq 0 \\ x_1 &\in [0.1, 0.2] \\ y_2, y_3 &\in \{1, 2, \dots, 200\} \end{aligned}$$

The best known optimal solution is $\mathbf{x}^* = 0.1365$ and $\mathbf{y}^* = (15, 3)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = -5.6848$.

F21 : minimize : $f(\mathbf{x}, \mathbf{y}) = 5.3578547x_1^2 + 0.8356891x_3y_4 + 37.293239y_4 - 40792.141$
subject to:

$$\begin{aligned} g_1(\mathbf{x}, \mathbf{y}) &= -(85.334407 - 0.0022053x_1x_3 + 0.0056858x_3y_5 + 0.0006262x_2y_4) \leq 0 \\ g_2(\mathbf{x}, \mathbf{y}) &= -(80.51249 + 0.0021813x_1^2 + 0.0071317x_3y_5 + 0.0029955y_4y_5 - 90) \leq 0 \\ g_3(\mathbf{x}, \mathbf{y}) &= -(9.300961 + 0.0019085x_1x_2 + 0.0047026x_1x_3 + 0.0012547x_1y_4 - 20) \leq 0 \\ g_4(\mathbf{x}, \mathbf{y}) &= 85.334407 - 0.0022053x_1x_3 + 0.0056858x_3y_5 + 0.0006262x_2y_4 - 92 \leq 0 \\ g_5(\mathbf{x}, \mathbf{y}) &= 80.51249 + 0.0021813x_1^2 + 0.0071317x_3y_5 + 0.0029955y_4y_5 - 110 \leq 0 \\ g_6(\mathbf{x}, \mathbf{y}) &= 9.300961 + 0.0019085x_1x_2 + 0.0047026x_1x_3 + 0.0012547x_1y_4 - 25 \leq 0 \\ x_1, x_2, x_3 &\in [27, 45] \\ y_4 &\in \{78, 79, \dots, 102\} \\ y_5 &\in \{33, 34, \dots, 45\} \end{aligned}$$

The best known optimal solution is $\mathbf{x}^* = (29.9953, 45, 36.7758)$ and $\mathbf{y}^* = (78, 33)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = -30665.5387$.

F22 : minimize : $f(\mathbf{x}, \mathbf{y}) = -\sum_{i=1}^3 x_i + 5 \sum_{i=4}^7 (y_i - y_i^2) - \sum_{i=8}^{13} y_i$
subject to:

$$\begin{aligned} g_1(\mathbf{x}, \mathbf{y}) &= x_1 + x_2 + 2y_4 + 2y_5 - 10 \leq 0 \\ g_2(\mathbf{x}, \mathbf{y}) &= x_1 + x_3 + 2y_4 + 2y_6 - 10 \leq 0 \\ g_3(\mathbf{x}, \mathbf{y}) &= x_2 + x_3 + 2y_5 + 2y_6 - 10 \leq 0 \\ g_4(\mathbf{x}, \mathbf{y}) &= x_1 - 8y_4 \leq 0 \\ g_5(\mathbf{x}, \mathbf{y}) &= x_2 - 8y_5 \leq 0 \\ g_6(\mathbf{x}, \mathbf{y}) &= x_3 - 8y_6 \leq 0 \\ g_7(\mathbf{x}, \mathbf{y}) &= x_1 - 2y_7 - y_8 \leq 0 \\ g_8(\mathbf{x}, \mathbf{y}) &= x_2 - 2y_9 - y_{10} \leq 0 \\ g_9(\mathbf{x}, \mathbf{y}) &= x_3 - 2y_{11} - y_{12} \leq 0 \\ x_1, x_2, x_3 &\in [0, 100] \\ y_4, y_5, \dots, y_{13} &\in \{0, 1\} \end{aligned}$$

The best known optimal solution is $\mathbf{x}^* = (3, 3, 3)$ and $\mathbf{y}^* = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = -15$.

F23 : minimize : $f(\mathbf{x}, \mathbf{y}) = (x_1 - 1)^2 + (x_2 - 2)^2 + (x_3 - 3)^2 + (y_4 - 1)^2 + (y_5 - 1)^2 + (y_6 - 1)^2 - \log(y_7 + 1)$

subject to:

$$\begin{aligned}
g_1(\mathbf{x}, \mathbf{y}) &= x_2 + y_5 - 1.8 \leq 0 \\
g_2(\mathbf{x}, \mathbf{y}) &= x_2^2 + y_5^2 - 1.64 \leq 0 \\
g_3(\mathbf{x}, \mathbf{y}) &= x_1 + x_2 + x_3 + y_4 + y_5 + y_6 - 5 \leq 0 \\
g_4(\mathbf{x}, \mathbf{y}) &= x_3 + y_6 - 2.5 \leq 0 \\
g_5(\mathbf{x}, \mathbf{y}) &= x_3^2 + y_6^2 - 4.25 \leq 0 \\
g_6(\mathbf{x}, \mathbf{y}) &= x_1^2 + x_2^2 + x_3^2 + y_6^2 - 5.5 \leq 0 \\
g_7(\mathbf{x}, \mathbf{y}) &= x_1 + y_7 - 1.2 \leq 0 \\
g_8(\mathbf{x}, \mathbf{y}) &= x_3^2 + y_5^2 - 4.64 \leq 0 \\
g_9(\mathbf{x}, \mathbf{y}) &= x_1 + y_4 - 1.2 \leq 0 \\
x_1, x_2, x_3 &\in [0, 1 \times 10^6] \\
y_4, y_5, y_6, y_7 &\in \{0, 1\}
\end{aligned}$$

The best known optimal solution is $\mathbf{x}^* = (0.2, 1.2806, 1.9545)$ and $\mathbf{y}^* = (1, 0, 0, 1)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 3.5575$.

$$\begin{aligned}
\mathbf{F24} : \text{minimize } f(\mathbf{x}, \mathbf{y}) &= 3(x_1 - 11)^2 + 10x_2^6 + 7x_3^2 + x_4^4 - 4x_3x_4 - 10x_3 - 8x_4 + (y_5 - 10)^2 \\
&\quad + 5(y_6 - 12)^2 + x_7^4
\end{aligned}$$

subject to:

$$\begin{aligned}
g_1(\mathbf{x}, \mathbf{y}) &= 4x_1^2 + 5x_2 + 2y_5^2 + 3y_6^4 + y_7 - 127 \leq 0 \\
g_2(\mathbf{x}, \mathbf{y}) &= x_1 - x_2 + 7y_5 + 3y_6 + 10y_7^2 - 282 \leq 0 \\
g_3(\mathbf{x}, \mathbf{y}) &= 6x_3^2 - 8x_4 + 23y_5 + y_6^2 - 196 \leq 0 \\
g_4(\mathbf{x}, \mathbf{y}) &= 5x_3 - 11y_4 + 4y_5^2 + y_6^2 - 3y_5y_6 + 2y_7^2 \leq 0 \\
x_1, x_2, x_3, x_4 &\in [-10, 10] \\
y_5, y_6, y_7 &\in \{-10, -9, \dots, 10\}
\end{aligned}$$

The best known optimal solution is $\mathbf{x}^* = (4.3339, -0.6260, 1.1323, 1.4632)$ and $\mathbf{y}^* = (2, 2, -1)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 682.8160$.

$$\mathbf{F25} : \text{minimize } f(\mathbf{x}, \mathbf{y}) = 0.6224x_1x_2y_3 + 1.7781x_1^2y_4 + 3.1661x_2y_3^2 + 19.84x_1y_3^2$$

subject to:

$$\begin{aligned}
g_1(\mathbf{x}, \mathbf{y}) &= 0.0193x_1 - y_3 \leq 0 \\
g_2(\mathbf{x}, \mathbf{y}) &= 0.00954x_1 - y_4 \leq 0 \\
g_3(\mathbf{x}, \mathbf{y}) &= 1296000 - \frac{4}{3}\pi x_1^3 - \pi x_1^2x_2 = 0 \\
x_1, x_2 &\in [10, 221.2] \\
y_3, y_4 &\in \{0.0625, 0.1250, \dots, 6.1875\}
\end{aligned}$$

The best known optimal solution is $\mathbf{x}^* = (42.0984, 176.6366)$ and $\mathbf{y}^* = (0.8125, 0.4375)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 6059.7143$.

$$\mathbf{F26} : \text{minimize } f(\mathbf{x}, \mathbf{y}) = \left(0.14427932477276 - \frac{x_1 \cdot x_2}{x_3 \cdot x_4} \right)^2$$

subject to:

$$h_1(\mathbf{x}, \mathbf{y}) = x_1 - y_5 = 0$$

$$\begin{aligned}
h_2(\mathbf{x}, \mathbf{y}) &= x_2 - y_6 = 0 \\
h_3(\mathbf{x}, \mathbf{y}) &= x_3 - y_7 = 0 \\
h_4(\mathbf{x}, \mathbf{y}) &= x_4 - y_8 = 0 \\
x_1, x_2, x_3, x_4 &\in [12, 60] \\
y_5, y_6, y_7, y_8 &\in \{12, 60\}
\end{aligned}$$

The best known optimal solution is $\mathbf{x}^* = (17, 15, 31, 57)$ and $\mathbf{y}^* = (17, 15, 31, 57)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 0$.

$$\begin{aligned}
\mathbf{F27} : \text{minimize} : f(\mathbf{x}, \mathbf{y}) &= 5.9207859 + 9.999999999999999 \times 10^{-05} \\
&\quad \times (5.3578547y_3^2 + 0.8356891y_1y_5 + 37.293239y_1)
\end{aligned}$$

subject to:

$$\begin{aligned}
h_1(\mathbf{x}, \mathbf{y}) &= -(0.0056858y_2y_5 + 0.0006262y_1y_4 - 0.0022053y_3y_5) \\
&\quad + x_6 - 85.334407 \\
h_2(\mathbf{x}, \mathbf{y}) &= -(0.0071317y_2y_5 + 0.0029955y_1y_2 + 0.0021813y_3^2) \\
&\quad + x_7 - 80.51249 \\
h_3(\mathbf{x}, \mathbf{y}) &= -(0.0047026y_3y_5 + 0.0012547y_1y_3 + 0.0019085y_3y_4) \\
&\quad + x_8 - 9.300961 \\
y_1, y_2, y_3, y_4, y_5 &\in \{1, 200\} \\
x_6 \in [0, 92], x_7 \in [90, 110], x_8 &\in [20, 25]
\end{aligned}$$

The best known optimal solution is $\mathbf{x}^* = (90.8476, 91.7561, 20.0271)$ and $\mathbf{y}^* = (1, 9, 9, 200, 172)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 5.9823$.

$$\mathbf{F28} : \text{minimize} : f(\mathbf{x}, \mathbf{y}) = 1.10471y_3^2y_4 + 0.04811y_1y_2(14 + y_4)$$

subject to:

$$\begin{aligned}
h_1(\mathbf{x}, \mathbf{y}) &= -\frac{4243.28147100424}{y_3y_4} + x_5 = 0 \\
h_2(\mathbf{x}, \mathbf{y}) &= -\sqrt{0.25y_4^2 + (0.5y_1 + 0.5y_3)^2} + x_7 = 0 \\
h_3(\mathbf{x}, \mathbf{y}) &= -\frac{(59405.9405940594 + 2121.64073550212y_4)x_7}{y_3y_4(0.0833333333333333y_4^2 + (0.5y_1 + 0.5y_3)^2)} + x_6 = 0 \\
h_4(\mathbf{x}, \mathbf{y}) &= -0.5\frac{y_4}{x_7} + x_8 = 0 \\
g_1(\mathbf{x}, \mathbf{y}) &= \sqrt{x_5^2 + 2x_5x_6x_8 + x_6^2} - 13600 \leq 0 \\
g_2(\mathbf{x}, \mathbf{y}) &= \frac{504000}{y_1^2y_2} - 30000 \leq 0 \\
g_3(\mathbf{x}, \mathbf{y}) &= -y_2 + y_3 \leq 0 \\
g_4(\mathbf{x}, \mathbf{y}) &= -0.0204744897959184\sqrt{1 \times 10^{13}y_2^3y_1^2y_2^3} \times (1 - 0.0282346219657891y_1) \\
&\quad + 6000 \leq 0 \\
g_5(\mathbf{x}, \mathbf{y}) &= \frac{2.1952}{y_1^3} - 0.25 \leq 0 \\
y_1, y_2 &\in \{1, 200\} \\
y_3, y_4 &\in \{1, 20\} \\
x_5, x_6, x_7, x_8 &\in [0, 1 \times 10^5]
\end{aligned}$$

The best known optimal solution is $\mathbf{x}^* = (2121.6408, 10782.9331, 3.1624, 0.3162)$ and $\mathbf{y}^* = (5, 1, 1, 2)$, and $f(\mathbf{x}^*, \mathbf{y}^*) = 6.0582$.

S-II. ADDITIONAL RESULTS

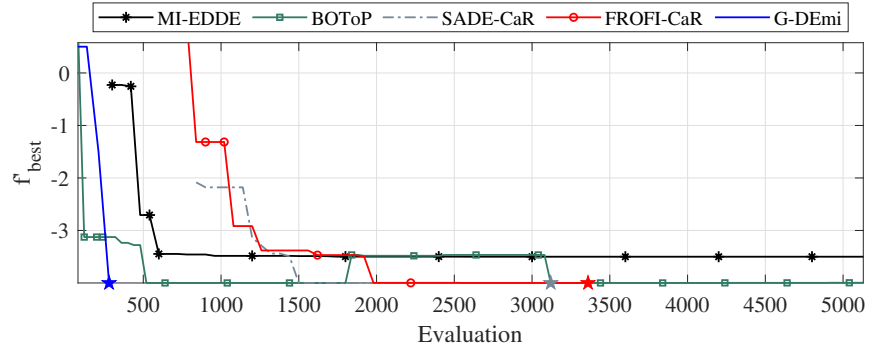


Figure S-I. Convergence curves of the median solutions from the 30 runs for problem F3.

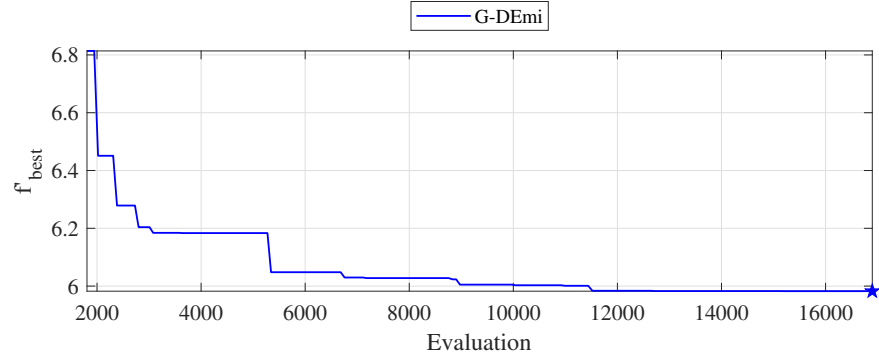


Figure S-II. Convergence curves of the median solutions from the 30 runs for problem F27. No other algorithm reached feasible solutions in the range of evaluations presented.

Table S-II. Average execution times in seconds and the average value of the objective function provided by G-DEmi on the F3 problem.

		$k_{max} = 0$	$k_{max} = 25$	$k_{max} = 50$	$k_{max} = 100$
$T_{min} = 1 \times 10^{-3}$	Ave.	-3.5833	-4.0000	-4.0000	-4.0000
	Time	3.4434	3.4672	3.3791	3.6942
$T_{min} = 1 \times 10^{-30}$	Ave.	-3.5833	-4.0000	-4.0000	-4.0000
	Time	3.3812	3.6974	3.6593	3.9220
$T_{min} = 1 \times 10^{-60}$	Ave.	-3.5667	-4.0000	-4.0000	-4.0000
	Time	3.4050	3.5872	3.5783	3.9204
$T_{min} = 1 \times 10^{-80}$	Ave.	-3.5500	-4.0000	-4.0000	-4.0000
	Time	3.4597	3.5580	3.5902	3.9860

Table S-III. Average execution times in seconds and the average value of the objective function provided by G-DEmi on the F12 problem.

		$k_{max} = 0$	$k_{max} = 25$	$k_{max} = 50$	$k_{max} = 100$
$T_{min} = 1 \times 10^{-3}$	Ave.	45.5244	41.7399	41.7450	41.7515
	Time	5.3546	23.1976	30.7741	43.9094
$T_{min} = 1 \times 10^{-30}$	Ave.	56.3351	41.7399	41.7450	41.7399
	Time	5.4118	28.6635	42.6500	69.8246
$T_{min} = 1 \times 10^{-60}$	Ave.	49.3507	41.7450	41.7399	41.7399
	Time	5.4309	28.1685	43.2747	73.8161
$T_{min} = 1 \times 10^{-80}$	Ave.	49.1143	41.7632	41.7399	41.7399
	Time	5.4355	27.0197	43.2668	71.2409

Table S-IV. Average execution times in seconds and the average value of the objective function provided by G-DEmi on the F27 problem.

		$k_{max} = 0$	$k_{max} = 25$	$k_{max} = 50$	$k_{max} = 100$
$T_{min} = 1 \times 10^{-3}$	Ave.	NA	5.9826	5.9823	5.9832
	Time	5.8850	6.8628	6.7666	6.8239
$T_{min} = 1 \times 10^{-30}$	Ave.	NA	5.9823	5.9823	5.9823
	Time	5.8674	5.8369	5.8406	6.0280
$T_{min} = 1 \times 10^{-60}$	Ave.	NA	5.9823	5.9823	5.9851
	Time	5.8153	5.8915	6.0963	6.5128
$T_{min} = 1 \times 10^{-80}$	Ave.	NA	5.9823	5.9823	5.9823
	Time	6.1649	5.8499	6.0117	6.3620

Table S-V. Performance comparison conducted over 30 independent runs with 200,000 objective function evaluations for PSOm_v, EDAm_v, and G-DEmi (Part 1 of 2).

Problem	Parameter	PSOm _v	EDAm _v	G-DEmi
F1	FR	100	100	100
	SR	0	96.67	100
	Ave ± Std	1.70E+01±0.00E+00 +	1.31E+01±7.30E-01 ≈	1.30E+01±7.23E-15
	T	4.83E+00	2.56E-01	3.60E+00
F2	FR	100	100	100
	SR	6.67	100	100
	Ave ± Std	2.87E+00±2.13E+00 +	1.00E+00±0.00E+00 ≈	1.00E+00±0.00E+00
	T	3.59E+01	5.05E-01	3.67E+00
F3	FR	100	100	100
	SR	0	100	100
	Ave ± Std	-3.57E+00±1.72E-01 +	-4.00E+00±0.00E+00 ≈	-4.00E+00±0.00E+00
	T	2.08E+01	3.92E-01	3.72E+00
F4	FR	100	100	100
	SR	100	100	100
	Ave ± Std	-6.00E+00±0.00E+00 ≈	-6.00E+00±0.00E+00 ≈	-6.00E+00±0.00E+00
	T	2.08E+01	3.81E-01	3.43E+00
F5	FR	100	100	100
	SR	6.67	67	100
	Ave ± Std	1.17E+00±2.51E-01 +	5.80E-01±4.75E-01 +	2.50E-01±2.54E-05
	T	7.71E+00	2.42E-01	3.48E+00
F6	FR	100	100	100
	SR	47	100	100
	Ave ± Std	-6.10E+03±7.47E+02 +	-6.78E+03±2.78E-12 ≈	-6.78E+03±2.78E-12
	T	2.07E+01	4.39E-01	3.60E+00
F7	FR	93.33	100	100
	SR	0	33.33	100
	Ave ± Std	NA	6.77E-01±3.61E-01 +	2.11E-01±1.13E-16
	T	1.80E+01	4.88E-01	7.45E+00
F8	FR	66.67	63.33	100
	SR	0	0	97
	Ave ± Std	NA	NA	7.06E+03±6.90E+00
	T	1.11E+02	1.21E+00	1.52E+01
F9	FR	36.67	56.67	100
	SR	0	0	100
	Ave ± Std	NA	NA	7.08E+03±0.00E+00
	T	6.23E+01	1.03E+00	8.72E+00
F10	FR	6.67	60	100
	SR	0	0	100
	Ave ± Std	NA	NA	7.13E+03±0.00E+00
	T	5.13E+01	9.20E-01	8.12E+00
F11	FR	100	100	100
	SR	0	0	60
	Ave ± Std	9.60E+01±6.64E+01 +	4.27E+01±6.82E+00 +	3.35E+01±4.95E-02
	T	2.82E+01	1.31E+00	5.49E+01
F12	FR	100	100	100
	SR	0	0	100
	Ave ± Std	1.98E+02±8.31E+01 +	9.11E+01±2.87E+01 +	4.17E+01±3.69E-13
	T	4.59E+01	1.25E+00	4.57E+01
F13	FR	100	100	100
	SR	0	0	100
	Ave ± Std	9.02E+03±9.72E+01 +	8.98E+03±8.16E+01 +	8.88E+03±5.47E-12
	T	2.12E+01	5.95E-01	1.60E+01
F14	FR	100	100	100
	SR	0	50	100
	Ave ± Std	9.12E+03±8.40E+01 +	8.98E+03±8.07E+01 +	8.95E+03±3.70E-12
	T	1.82E+01	5.91E-01	6.87E+00
F15	FR	100	100	100
	SR	0	0	96.67
	Ave ± Std	4.29E+01±9.54E+00 +	3.38E+01±3.42E+00 +	2.84E+01±7.53E-02
	T	3.25E+01	7.75E-01	1.16E+01

Table S-VI. Performance comparison conducted over 30 independent runs with 200,000 objective function evaluations for PSOm_v, EDAm_v, and G-DEm_i (Part 2 of 2).

Problem	Parameter	PSOm _v	EDAm _v	G-DEm _i
F16	FR	100	100	100
	SR	0	0	100
	Ave ± Std	6.84E+01±4.21E+01 +	4.44E+01±1.41E+01 +	2.85E+01±9.67E-15
	T	5.23E+01	7.23E-01	9.60E+00
F17	FR	0	36.67	100
	SR	0	0	100
	Ave ± Std	NA	NA	6.01E+00±2.71E-15
	T	2.34E+01	5.60E-01	5.04E+00
F18	FR	100	100	100
	SR	0	57	100
	Ave ± Std	-2.17E+01±2.86E-02 +	-2.17E+01±1.21E-02 +	-2.17E+01±1.72E-08
	T	7.91E+01	1.26E+00	1.51E+01
F19	FR	0	0	100
	SR	0	0	100
	Ave ± Std	NA	NA	9.92E+01±0.00E+00
	T	1.60E+01	5.81E-01	8.17E+00
F20	FR	100	100	100
	SR	0	43.33	100
	Ave ± Std	-4.87E+00±4.06E-01 +	-5.24E+00±4.24E-01 +	-5.68E+00±9.03E-16
	T	4.87E+01	6.13E-01	3.85E+00
F21	FR	100	100	100
	SR	93.33	100	100
	Ave ± Std	-3.07E+04±1.24E+01 ≈	-3.07E+04±1.89E-09 ≈	-3.07E+04±1.44E-11
	T	2.23E+01	4.88E-01	4.43E+00
F22	FR	100	100	100
	SR	100	100	100
	Ave ± Std	-1.50E+01±0.00E+00 ≈	-1.50E+01±1.12E-09 ≈	-1.50E+01±0.00E+00
	T	7.25E+01	7.21E-01	4.10E+00
F23	FR	100	100	100
	SR	23.33	10	93.33
	Ave ± Std	4.24E+00±5.39E-01 +	4.49E+00±4.27E-01 +	3.56E+00±5.61E-03
	T	3.03E+01	4.95E-01	5.00E+00
F24	FR	100	100	100
	SR	20	60	100
	Ave ± Std	6.85E+02±4.16E+00 +	6.83E+02±1.87E+00 +	6.83E+02±6.68E-14
	T	3.38E+01	6.60E-01	6.22E+00
F25	FR	100	60	100
	SR	0	26.67	100
	Ave ± Std	8.25E+03±1.01E+03 +	NA	6.06E+03±9.25E-13
	T	4.47E+01	6.40E-01	4.20E+00
F26	FR	100	100	100
	SR	0	13.33	100
	Ave ± Std	7.84E-01±2.72E-01 +	4.54E-02±1.05E-01 +	1.39E-07±2.71E-07
	T	6.03E+01	8.80E-01	5.68E+00
F27	FR	100	100	100
	SR	0	0	100
	Ave ± Std	8.25E+00±5.33E-01 +	7.55E+00±5.61E-01 +	5.98E+00±0.00E+00
	T	1.44E+02	1.26E+00	5.70E+00
F28	FR	100	100	100
	SR	0	0	100
	Ave ± Std	6.38E+03±2.10E+03 +	7.35E+03±2.66E+03 +	6.06E+00±2.71E-15
	T	9.00E+01	1.18E+00	8.74E+00
Mean FR		85.83	88.45	100
Mean SR		14.17	37.74	98.10
WRST [+/-]		[25/3/0]	[21/7/0]	

S-III. PARAMETERS OF SUBWAY OPTIMIZATION PROBLEM

Table S-VII. Parameters used in SOP

Parameter	Value
Weight of the train, [lbs]	$W = 78,000$
Effective weight of the train, [lbs]	$W_{eff} = 85,200$
Scaling factor for units, [s/h/ft/mile]	$\gamma = 3600/5280$
Front surface of the train, [mile ²]	$a = 100/(5280)^2$
Drag formula parameter, adim	$b = 0.045$
Drag formula parameter, adim	$c = 0.240306$
Constant braking when coasting, adim	$C = 0.367$
Gravity, [ft/s ²]	$g = 32.2$
Percentage of working machines, adim	$e = 1$
Maximal deceleration, [ft/s ²]	$u_{max} = 4.4$

Table S-VIII. Operation Modes Dependent Parameters.

Parameter	Value
Velocity limits, [mph]	$v_1=0.979474$
	$v_2=6.73211$
	$v_3=14.2658$
Accelerations, [lbs]	$a_1=6017.611205$
	$a_2=12348.34865$
	$a_3=11124.63729$
Energy consumption	$p_1=106.1951102$
	$p_2=180.9758408$
	$p_3=354.136479$

Table S-IX. Coefficients used in SOP

$b_0(1) = -0.1983670410 \times 10^2$	$c_0(1) = 0.3629738340 \times 10^2$
$b_1(1) = 0.1952738055 \times 10^3$	$c_1(1) = -0.2115281047 \times 10^3$
$b_2(1) = 0.2061789974 \times 10^4$	$c_2(1) = 0.7488955419 \times 10^3$
$b_3(1) = -0.7684409308 \times 10^3$	$c_3(1) = -0.9511076467 \times 10^3$
$b_4(1) = 0.2677869201 \times 10^3$	$c_4(1) = 0.5710015123 \times 10^3$
$b_5(1) = -0.3159629687 \times 10^2$	$c_5(1) = -0.1221306465 \times 10^3$
$b_0(2) = -0.1577169936 \times 10^3$	$c_0(2) = 0.4120568887 \times 10^2$
$b_1(2) = 0.3389010339 \times 10^4$	$c_1(2) = 0.3408049202 \times 10^3$
$b_2(2) = 0.6202054610 \times 10^4$	$c_2(2) = -0.1436283271 \times 10^3$
$b_3(2) = -0.4608734450 \times 10^4$	$c_3(2) = 0.8108316584 \times 10^2$
$b_4(2) = 0.2207757061 \times 10^4$	$c_4(2) = -0.5689703073 \times 10^1$
$b_5(2) = -0.3673344160 \times 10^3$	$c_5(2) = -0.2191905731 \times 10^1$

S-IV. TIME COMPLEXITY

As a complement to the reported execution times, the algorithm's complexity is analyzed below. However, this analysis is more of a general estimate than a mathematically accurate formula. Big-O notation provides an upper limit on the algorithm's running time, indicating the number of times each main operation runs. It represents the worst-case complexity of an algorithm with a function $O(f(n))$.

Differential evolution consists of usual operators (mutation, crossover, selection). Its complexity is $O(g(nm + nm + n))$, where g is the number of generations, n is the population size, and m is the variables per individual. The previous analysis reveals that the complexity of mutation and crossover is proportional to g , m , and n . In contrast, the selection process is not affected by the length of the solution vector.

Once the gradient-based repair method is included, the principal operators change to mutation, crossover, selection, and repair. During the repair process, we have two main processes: calculating $\nabla \mathbf{V}$ and computing the inverse of $\nabla \mathbf{V}$. Since the number of variables is often higher than the number of constraints, $\nabla \mathbf{V}$ can be considered of dimensions m . Therefore, the complexity of $\nabla \mathbf{V}$ is $O(m)$, and the complexity of the $\nabla \mathbf{V}$ inversion is $O(m^3)$, making the matrix inversion the dominant operation. The worst-case scenario for the repair process involves addressing all individuals in the population over K_{max} iterations. Since our proposal focuses on MINLP problems, the solution vector includes at least one integer variable. Hence, the repair method involves $n(m-1)^3$ operations repeated K_{max} times, and the complexity is $O(nK_{max}(m-1)^3)$. Then, the overall complexity expression is modified as $O(g(nm + nm + n + nK_{max}(m-1)^3))$. After removing negligible terms, the expression can be reduced to $O(gnm^3K_{max})$.