

Supplement 1

Glossary of Notation and Abbreviations

- a : area searched during line-transect survey, where $a = 2Lw$, L is the total length of transects surveyed, and w is the width of the strip searched on one side of the aircraft
- \mathbf{a} : vector of the area surveyed for each transect segment ($a_i = 2L_iw$)
- A : total study area size
- \mathbf{A} : an (n_i, n_η) interpolation matrix used to convert raw random effects into transect-specific values. For SPDE models, this is constructed with a Delauney triangulation. For basis-penalty smooth models, \mathbf{A} is simply the design matrix associated with spatial smooth parameters.
- \mathbf{A}^{pred} : an (n_h, n_η) interpolation matrix used to convert raw random effects into grid-cell specific predictions. For SPDE models, this is constructed with a Delauney triangulation. For spline-based smoother models, \mathbf{A} is a design matrix associated with locations of grid cell centroids (obtained using the “predict” function in `mgcv`).
- \mathbf{c} : vector of observed counts of individual animals
- c_i : observed number of individual animals on transect segment i
- $[\mathbf{c}|\xi, \boldsymbol{\eta}, \mathbf{x}]$: conditional probability density function of observed counts, given parameters, random effects, and known covariates
- CV : coefficient of variation
- \hat{D} : estimate of density of animals (number of animals per unit area)
- DSM : density surface model
- EBS belugas : Eastern Bering Sea belugas
- $g(y_j, \mathbf{z}_j; \hat{\boldsymbol{\theta}}_g)$: probability of detecting an animal at distance y_j , given that it is available to be seen and is associated with covariates \mathbf{z}_j , assuming perfect detection on the transect
- h : grid cell index
- i : segment index
- j : group index
- k : bootstrap replicate index
- L : transect length
- n_g : number of groups detected
- n_i : number of transect segments in DSM
- n_h : number of grid cells in DSM
- n_η : number of random effects in DSM
- \hat{N} : estimate of the total number of animals in the study area
- $\hat{p}(\mathbf{z}_j; \hat{\boldsymbol{\theta}})$: model-based estimate of the overall probability that an observer detects a group of whales, given covariates \mathbf{z}_j that affect detectability. This term accounts for all sources of perception and availability bias (Marsh and Sinclair 1989; S4).

$\hat{p}_g(\mathbf{z}_j; \hat{\boldsymbol{\theta}}_g)$: average probability that an observer detects an object that is available to be seen in the area searched, given covariates \mathbf{z}_j that affect detectability, assuming transect detection probability is 1.0
 \mathbf{p}_g : vector of $\hat{p}_g(\mathbf{z}_j; \hat{\boldsymbol{\theta}}_g)$ for a collection of sightings indexed by j
 p_i : shorthand for $\hat{p}(\mathbf{z}_i; \hat{\boldsymbol{\theta}})$ for transects. Note that making the change from group-level detection probability (subscript j) to transect-level detect probability (subscript i) requires that we omit group-specific covariates, such as group size.
 \mathbf{p}_i : vector of the overall detection probability (including both availability and perception bias corrections) for each segment, p_i .
 p_j : shorthand for $\hat{p}(\mathbf{z}_j; \hat{\boldsymbol{\theta}})$ for groups
 $p^*(y_j, \mathbf{z}_j; \hat{\boldsymbol{\theta}})$: probability of detecting an animal at distance y_j , given that it is associated with covariates \mathbf{z}_j
 \hat{p}_A : estimate of availability probability, defined as the probability that a group is at the surface within an observer's field of view
 $\hat{p}_{MR}(0, \mathbf{z}_j; \hat{\boldsymbol{\theta}}_{MR})$: estimate of transect detection probability, defined as the probability of detecting an animal on the transect (or left-truncation point, if applicable)
 \mathbf{s} : vector of knot locations for SPDE models
 S_j : size of group indexed by j
 SPDE : stochastic partial differential equation
 \mathbf{Q} : precision (inverse covariance) matrix for random effects
 Var : variance
 w : distance (width) searched on one side of the transect
 \mathbf{x} : vector of known covariates used in the DSM
 y_j : perpendicular distance from the transect line to the sighting of group j
 \mathbf{z}_i : covariates that affect detectability on segment i
 \mathbf{z}_j : covariates that affect detectability of group j
 β_0 : DSM intercept parameter
 $\boldsymbol{\delta}$: vector of 'realized' random effects for transect counts
 $\boldsymbol{\eta}$: vector of random effects for the DSM
 $[\boldsymbol{\eta}|\mathbf{x}, \boldsymbol{\xi}]$: probability density function of random effects for the DSM
 $\hat{\boldsymbol{\theta}}$: parameter estimates required to estimate detection probabilities
 μ : mean of a Tweedie probability density function
 μ_i : expected number of whales encountered on transect segment i
 $\boldsymbol{\xi}$: vector of unknown parameters for the DSM
 ρ : power parameter for the Tweedie probability density function
 ϕ : dispersion parameter for the Tweedie probability density function
 τ : Matérn precision parameter
 κ : Matérn inverse range parameter
 λ : penalization parameter(s) for basis-penalty smooths
 $\lambda_{m,h}$: predicted abundance from model m for unsampled location h
 $\lambda_{m,max}$: maximum predicted abundance across all sampled cells